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AN ADAPTIVE KALMAN IDENTIFIER  
AND ITS APPLICATION TO  
LINEAR AND NON-LINEAR ARMA MODELING

Leopoldo M. Mayoral



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

AN ADAPTIVE KALMAN IDENTIFIER  
AND ITS APPLICATION TO  
LINEAR AND NON-LINEAR ARMA MODELING

by

Leopoldo M. Mayoral

March 1981

Thesis Advisor:

S.R. Parker

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With minor modification, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.

## #19 - KEY WORDS - (CONTINUED)

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An Adaptive Kalman Identifier  
and Its Application to  
Linear and Non-Linear ARMA Modeling

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requirements for the degree of

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## ABSTRACT

The problem of accurately replicating the parameters which define a given system for the purposes of implementing modern control strategies is important. Using an Autoregressive-Moving Average (ARMA) representation for the unknown system, a model is identified by processing input/output data to estimate the coefficients associated with the ARMA equation. Identification of unknown system parameters using Kalman filtering methods was accomplished by augmenting the state vector. In this thesis the Kalman filter is formulated so that parameters can be identified explicitly. We call this approach the Adaptive Kalman Identifier (AKI).

It is shown that the Adaptive least mean square (LMS), Adaptive Recursive LMS and Adaptive Lattice filters are special suboptimal cases of the AKI. The convergence and modeling properties are compared with those of the AKI by simulation using various types of data.

With minor modifications, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.



## TABLE OF CONTENTS

I.	INTRODUCTION -----	13
	A. BACKGROUND -----	13
	B. INVESTIGATIONS AND CONTRIBUTIONS -----	16
	C. ORGANIZATION -----	17
II.	FORMULATION OF THE PROBLEM -----	18
	A. DETERMINISTIC CASE -----	18
	B. NON DETERMINISTIC CASE -----	20
	1. Autoregressive Form ( $a_i = 0$ ) -----	24
	2. Moving Average Form ( $b_i = 0$ ) -----	27
	3. Autoregressive-Moving Average Form -----	30
	C. OBSERVATIONS -----	33
III.	COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND WIDROW LMS ADAPTIVE FILTER -----	36
	A. PRELIMINARIES -----	36
	B. THE COMPARISON -----	38
	C. OBSERVATIONS -----	41
IV.	COMPARISON BETWEEN THE ADAPTIVE KALMAN IDEN- TIFIER AND FEINTUCH ADAPTIVE RECURSIVE LMS FILTER -----	43
	A. PRELIMINARIES -----	43
	B. THE COMPARISON -----	47
	C. OBSERVATIONS -----	53
V.	SYNTHETIC DATA GENERATING PLANTS -----	54
	A. AUTOREGRESSIVE-MOVING AVERAGE DATA -----	54
	1. Moving Average Data (Case 1) -----	55
	2. Autoregressive Data (Case 2) -----	55
	3. Autoregressive-Moving Average Data (Case 3) -----	56



B.	PHASE LOCKED LOOP DATA -----	57
VI.	SOFTWARE -----	67
A.	ADAPTIVE KALMAN IDENTIFIER -----	67
B.	LMS ADAPTIVE FILTER -----	69
C.	ADAPTIVE RECURSIVE LMS FILTER -----	73
VII.	NON-LINEAR IDENTIFICATION -----	75
VIII.	FINDINGS, CONCLUSION -----	81
A.	ORDER OF THE UNKNOWN SYSTEM IS KNOWN -----	81
1.	AKI vs LMS Adaptive Filter -----	81
2.	AKI Applied to Autoregressive Data -----	91
3.	AKI Applied to ARMA Data -----	93
4.	AKI vs Adaptive Recursive LMS Filter ----	98
5.	AKI Applied to PLL Data (Linear Region) -	103
6.	AKI Applied to PLL Data (Non-linear Region) -----	103
B.	ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING) -----	109
1.	AKI vs LMS Adaptive Filter -----	109
2.	AKI Applied to AR and ARMA Data -----	114
C.	CONCLUSION -----	115
D.	TOPICS FOR FURTHER CONSIDERATION -----	118
APPENDIX A:	The Discrete Weiner Problem -----	120
APPENDIX B:	Program ARMA -----	124
APPENDIX C:	Program PLL -----	126
APPENDIX D:	Program ADAPTSN -----	131
APPENDIX E:	Program LMS -----	147
APPENDIX F:	Program LMSR -----	151





LIST OF REFERENCES -----	158
INITIAL DISTRIBUTION LIST -----	161



## LIST OF TABLES

### TABLE

6.1	AKI Flag Meanings -----	68
6.2	Valid Flag Combinations -----	69
7.1	Non-linear Weighting Coefficients Calculated Using Three Methods -----	80
8.1	Actual vs AKI Estimates for Coefficients of Equation 8.3 -----	93
8.2	Actual vs AKI Estimates for Coefficients of Equation 8.4 at the 371st Iteration -----	94
8.3	Actual vs AKI and Adaptive Recursive LMS Filter Estimates for Coefficients of Equation 8.6 -----	100
8.4	Actual vs AKI Estimates of the ARMA Repre- sentation for the PLL (Linear Region) -----	103
8.5	Overmodeling of a MA Process Using the AKI ( $k = 250$ ) -----	113
8.6	Overmodeling of a MA Process Using the AKI of orders $AR = 2$ , $MA = 6$ -----	113



## LIST OF FIGURES

### FIGURE

2.1	Adaptive Kalman Identifier -----	34
3.1	LMS Adaptive Filter Implementation of System Identification -----	37
4.1	Adaptive Recursive LMS Filter Applied to System Identification -----	48
5.1	ARMA Digital Plant -----	56
5.2	PLL Block Diagram -----	58
5.3	Final PLL Block Diagram -----	59
5.4	PLL Step Response, Linear Region -----	60
5.5	PLL Step Response, Non-linear Region -----	61
5.6	Discrete PLL Block Diagram -----	62
5.7	Programmable PLL Implementing Forward Euler Integration -----	63
5.8	Discrete PLL Step Response, Linear Region ----	64
5.9	Discrete PLL Step Response, Non-linear Region -----	65
6.1	AKI General Program Flow -----	70
6.2	LMS Filter Program Flow -----	72
6.3	LMSR Filter Program Flow -----	74
7.1	PLL, Third Order Taylor Series Approximation -	76
8.1	LMS Adaptive Filter Weights, $k_s = -.200$ -----	83
8.2	LMS Adaptive Filter Weights, $k_s = -.300$ -----	84
8.3	LMS Adaptive Filter Weights, $k_s = -.400$ -----	85
8.4	LMS Adaptive Filter Weights, $k_s = -.500$ -----	86
8.5a	LMS Adaptive Filter Weights, $k_s = -.600$ -----	87





# FIGURE

8.5b	LMS Adaptive Filter Weights, $k_s = -.200$ , with Increased Measurement Noise -----	88
8.6	AKI Moving Average Plant Coefficients -----	89
8.7	LMS Adaptive Filter Weights -----	89
8.8	AKI Averaged Gains for Moving Average Model --	90
8.9	AKI Moving Average Instantaneous Gains -----	90
8.10	AKI Autoregressive Plant Coefficients -----	92
8.11	AKI Autoregressive Averaged Gains -----	92
8.12	AKI(4,5) Computed Coefficients, Perry's Example -----	95
8.13	AKI(4,5) Average Gain Values -----	95
8.14a	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) ----	96
8.14b	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) at the 371st Iteration -----	97
8.15	Instantaneous Gain Values, AKI(4,5) Model ----	98
8.16	AKI(2,2) Computed Coefficients, Feintuch Example -----	101
8.17	LMSR Computed Coefficients, Feintuch Example -	101
8.18	AKI(2,2) Averaged Gains, Feintuch Example ----	102
8.19	Pole/zero Models Produced by AKI and Adaptive Recursive LMS Algorithms for a Plant with the Characteristic Transfer Function of Equation 8.6 -----	102
8.20	PLL, AKI(2,3) Computed Coefficients -----	104
8.21	Averaged Gain Values, AKI(2,3) PLL Model ----	104
8.22	PLL Root Locus Analysis vs Computed Roots of the AKI ARMA Model -----	106



# FIGURE

8.23	Variation of $\bar{\alpha}$ vs Input Noise Signal Power ---	108
8.24	Comparison of $\frac{\sin x}{x}$ and P.D.F. of Input Noise Signal -----	110
8.25a	AKI(0,4) Model of a Third Order MA Process ----	112
8.25b	Fourth Order LMS Filter Weights of a Third Order MA Process -----	112
8.26	AKI(3,1) Overmodeled AKI(4,2) -----	116
8.27	AKI(3,1) Overmodeled AKI(5,5) -----	116
8.28	AKI(4,5) Overmodeled AKI(5,6) -----	117
A.1	The Discrete Weiner Problem -----	121



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## I. INTRODUCTION

The accomplishments in the area of microprocessor technology in the last decade have made a noticable impact in the area of modern control. The desire to implement modern control theories taking advantage of these advancements, has made it imperative that the engineer attempt to mathematically replicate the system he ultimately desires to control. Efforts in this regard have, hence, generated a growing interest in the area of system identification [Ref. 1,2,3,4]. By implication this thesis concerns itself with discrete/digital signal processing.

### A. BACKGROUND

System identification or modeling can be accomplished by innovative application of existing techniques which were generally considered filtering or state estimation methods [Ref. 5]. Previous research efforts have, for obvious reasons, focused on linear modeling; however, there is a rising interest in non-linear modeling methods [Ref. 6,7,8].

An attractive form for the representation of an unknown system is the Autoregressive-Moving Average (ARMA) equation,

$$y(k) = \sum_{i=0}^{\infty} a_i u(k-i) - \sum_{i=1}^{\infty} b_i y(k-i) \quad (1.1)$$

which states that the present output,  $y(k)$ , is a linear combination of past outputs,  $y(k-i)$ , and of past and present



inputs,  $u(k)$ . Its attractiveness lies in its linear character and easily implementable structure using microprocessor/computer algorithms. Its relationship to the state space representation of a linear system has already been established [Ref. 9,10] making it germane to consider that a system is identifiable by an equation of the form (1.1). The system identification problem thus entails identifying the coefficients  $a_i$  and  $b_i$ .

There are several methods for computing the coefficients,  $a_i$  and  $b_i$ , however, it is not the intent of this brief introduction to attempt to develop even the majority of them. Nevertheless, it is practical to present a referenced history of those methods encountered in this thesis.

Adaptive algorithms for the purpose of estimating the coefficients of (1.1) have always been of interest. Widrow, using a least means square error criterion and implementing the method of steepest descent, developed an adaptive algorithm which estimated the coefficients of the moving average process associated with equation (1.1) [Ref. 11]. That is, the moving average model,

$$y(k) = a'_0 u(k) + a'_1 u(k-1) + a'_2 u(k-2) + \dots \quad (1.2)$$

where,

$$a'_0 = a_0$$

$$a'_1 = (a_1 - a_0 b_1)$$



$$\begin{aligned} a'_2 &= (a_2 - a_0 b_2) - b_1(a_1 - a_0 b_1) \\ &\vdots \\ &\vdots \end{aligned}$$

was identified. The LMS theory has since been extended to include block LMS filtering methods [Ref. 12,13,14] using various search techniques [Ref. 15,4(Chapter 5)]. These methods have enjoyed much popularity in the area of Linear Prediction and digital speech processing [Ref. 16,17].

The shortcoming of representing equation (1.1) by its equivalent moving average model (1.2) lies in the practical aspect of its implementation. That is, the infinite series represented by equation (1.2) must be truncated at some point resulting in an approximation which may not adequately represent equation (1.1). Hence, efforts to adaptively estimate the coefficients for both the autoregressive ( $b_i$ ) and moving average ( $a_i$ ) processes continued [Ref. 18,19,20] with various degrees of success. Feintuch's Adaptive Regressive LMS procedure [Ref. 18] is still a controversial issue [Ref. 19,20].

From yet a different direction, Anderson and Moore suggested that the Kalman filtering algorithms can be used as a means to identify the  $a_i$  and  $b_i$  coefficients<sup>1</sup> of equation (1.1) [Ref. 21, pp. 50-52]. This thesis exploits this application.

---

<sup>1</sup>Anderson and Moore in fact formulate the technique to compute the coefficients  $a_i, b_i$   $i = 1, 2, \dots, p$  where  $p = n+m$ .



## B. INVESTIGATIONS AND CONTRIBUTIONS

This thesis formulates an adaptive application of the Kalman filter to identify the coefficients of the general ARMA equation (1.1). It is shown that the LMS adaptive filter [Ref. 11] and the Adaptive Recursive LMS filter [Ref. 18] are (1) special cases of the Adaptive ARMA Kalman identifier and (2) sub-optimal with respect to the underlying least means square (LMS) error criterion upon which the LMS adaptive filter, the adaptive recursive filter and the Kalman filter are based. It is also demonstrated that the adaptive Kalman identifier has excellent convergence properties.

By comparison, it is noted that the Kalman algorithm accounts for measurement noise where the LMS algorithms do not, a heretofore unapproached problem by LMS algorithms. The results indicate that the suggested modification by Griffiths [Ref. 22] of the convergence factor,  $k_s$ , for the LMS adaptive filter is justified.

The application of the Kalman filter algorithm is extended to identification of the coefficients associated with a special case of the generalized non-linear ARMA model [Ref. 8]. The results of the non-linear simulations suggest a technique for determining when a potential non-linear system enters its non-linear operating regime. Such a technique can prove valuable when on-line performance evaluation of a known non-linear system is required.

Lastly, the connection between the Kalman filter algorithm and the lattice filter algorithm [Ref. 6] is made. An example





which demonstrates and compares the performance of both algorithms is given.

### C. ORGANIZATION

Chapter II presents and exploits the Kalman filter equations emphasizing its connections with the Yule-Walker and the discrete Weiner-Hopf equations. The theory is further developed to investigate the general ARMA case. Chapter III pursues the theoretical comparisons between the discrete Weiner-Hopf equation upon which the adaptive LMS filter is based and the MA form of the Kalman filter. The theoretical comparison between the Adaptive Recursive LMS filter and the general ARMA form of the Kalman filter is made in Chapter IV. The software methods by which linear and non-linear synthetic data is generated are discussed in Chapter V. A short user's description of the data processing programs and the options provided is given in Chapter VI. A non-linear application for the identification of the parameters of a non-linear plant is presented in Chapter VII followed by a discussion and presentation of the results in Chapter VIII.



## II. FORMULATION OF THE PROBLEM

Identifying the coefficients of the Autoregressive-Moving Average (ARMA) equation,

$$y(k) + b_1 y(k-1) + \dots + b_n y(k-n) = a_0 u(k) + a_1 u(k-1) + \dots + a_m u(k-m) \quad (2.1a)$$

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) \quad (2.1b)$$

can be formulated as an adaptive Kalman identification problem. That is, instead of using the well known Kalman Filter equations [Ref. 23,24] to recursively estimate the states of a system, one can utilize the Kalman filter equations to adaptively estimate the coefficients of either an Autoregressive (AR), Moving Average (MA), or Autoregressive-Moving Average (ARMA) process by proper definition of the quantities involved. We call this the Adaptive Kalman Identifier for obvious reasons.

### A. THE DETERMINISTIC CASE

Consider for the moment that the  $a_i$  and  $b_i$  are constant. Then by collecting a sufficient number of measurements of the input  $u(k)$  and the output  $y(k)$ , one can readily solve a set of linear equations for the  $a_i$  and  $b_i$  coefficients. The "sufficient number" that is needed is  $n+m+1$  which define the



$n+m+1$  linear equations which solve the  $n+m+1$  unknown coefficients. The matrix equation to be solved takes the form:

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(m) \\ y(m+1) \\ \vdots \\ y(m+n) \end{bmatrix} = \begin{bmatrix} u(0) & 0 & \dots & 0 & 0 & \dots & \dots & \dots & 0 \\ u(1) & u(0) & & & y(0) & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ u(m) & u(m-1) & & u(0) & y(m-1) & y(m-2) & y(0) & & 0 \\ \hline u(m+1) & & & u(1) & y(m) & y(m-1) & y(1) & & 0 \\ \vdots & & & & & & & & \vdots \\ u(m+n) & \dots & & u(m) & y(n-1) & y(n-2) & & & y(0) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \\ b_1 \\ b_2 \\ \vdots \end{bmatrix} \quad (2.2)$$

$$\underline{y} = T \begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} \quad (2.3)$$

It is interesting to note that the  $(n+m+1) \times (n+m+1)$  matrix,  $T$ , is block symmetric. The solution to (2.3) is readily apparent, namely,

$$\begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} = T^{-1} \underline{y} \quad (2.4)$$

where  $T^{-1}$  is the inverse matrix of  $T$ . Since we have assumed that the elements of  $T$  are perfect noiseless measurements, and that we somehow know a-priori the number of unknown



coefficients, then  $T$  is of full rank, namely  $\text{rank}(T) = n+m+1$ , and the inverse of  $T$  exists.

#### B. THE NONDETERMINISTIC CASE

Rarely can the coefficients be modeled so ideally. A more prudent and realistic model admits that the ARMA equation coefficients are subject to random perturbations. Further, it can be said that in general measurement devices introduce noise into the measurement data. Hence, the measurement model should reflect this fact. Developing the Adaptive Kalman Identifier along the same lines as [Ref. 21] we let,

$$a_i(k+1) = a_i(k) + w_i(k) \quad i = 0, 1, 2, \dots, m \quad (2.5a)$$

$$b_j(k+1) = b_j(k) + w_j(k) \quad j = 1, 2, \dots, n \quad (2.5b)$$

where the  $\{w_i(k), w_j(k)\}$  are samples from a zero mean, white, gaussian random process. Additionally, we assume that the noise sources are uncorrelated,

$$E[w_{a_r}(k)w_{a_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6a)$$

$$E[w_{b_r}(k)w_{b_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6b)$$

$$E[w_{a_r}(k)w_{b_s}(k)] = 0 \quad \text{for all } r, s \quad (2.6c)$$

Similarly, allowing for noisy measurement devices, the measurement equation (2.1b), is modelled as,

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) + v(k) \quad (2.7)$$





where the  $\{v(k)\}$  are samples from a zero mean, white, gaussian random process. It is also assumed that,

$$E[w_i(k) v(k)] = 0 \quad \text{for all } i \quad (2.8a)$$

$$E[w_j(k) v(k)] = 0 \quad \text{for all } j \quad (2.8b)$$

That is, it is assumed that the noise perturbations associated with one coefficient are independent of the noise perturbations associated with any other coefficient, and that the measurement noise and the coefficient perturbations are independent.

At this point it is necessary to use judgment and experience and utilize all the information known about the physical system which equation (2.7) represents to assign variances for the random processes  $\{w_i(k)\}$ ,  $\{w_j(k)\}$  and  $\{v(k)\}$ . Let

$$Q = E \left\{ \begin{bmatrix} w_i(k) \\ w_j(k) \end{bmatrix} [w_i(k) \ w_j(k)] \right\} \quad (2.9a)$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \quad (2.9b)$$

where,  $Q_1 = \text{diag} (\sigma_{w_i}^2)$ ,  $Q_2 = \text{diag} (\sigma_{w_j}^2)$  and,

$$R = E[v(k) v(k)] \quad (2.9c)$$

and  $E[x]$  is the expected or average value of  $x$ .



Instead of using  $x$ 's denoting the commonly used notation for state variables, we retain the flavor of the problem by using the  $a_i$ 's and  $b_i$ 's as the "states" of the Adaptive Kalman Identifier. It is hoped that a more meaningful understanding of the Adaptive Kalman Identifier may thus be gained. Therefore, define the state vector,

$$\begin{bmatrix} a_0(k) \\ a_1(k) \\ \vdots \\ a_m(k) \\ \hline b_1(k) \\ b_2(k) \\ \vdots \\ b_n(k) \end{bmatrix} = \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} \quad (2.10)$$

Combining equations (2.5) and (2.10), we have the discrete, first order Gauss-Markov process [Ref. 3],

$$\begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} + \underline{w}(k) \quad (2.11)$$

where,

$$\underline{w}(k) = \begin{bmatrix} \underline{w}_1(k) \\ \hline \underline{w}_j(k) \end{bmatrix} . \quad (2.12)$$



Note that  $\underline{w}(k)$  is an  $n+m+1$  vector whose elements are a concatenation of the noise sequences associated with the  $a_i$  and  $b_i$ . To complete the Adaptive Kalman Identifier formulation we define the measurement vector,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m) \ -y(k-1) \ \dots \ -y(k-n)] \quad (2.13)$$

and its associated measurement equation,

$$y(k) = H(k) \begin{bmatrix} \underline{a}(k) \\ \text{-----} \\ \underline{b}(k) \end{bmatrix} + v(k). \quad (2.14)$$

Then the solution to the Adaptive Kalman Identifier problem [Ref. 21] is,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \text{-----} \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} = [I - K(k)H(k)] \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \text{-----} \\ \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k)y(k) \quad (2.15a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (2.15b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.15c)$$

Equations (2.15) are initialized by assuming an initial value for the coefficients (2.10) and assigning to our assumption a measure of our confidence in the initial guess. That is, we pick the values,

$$\begin{bmatrix} \hat{\underline{a}}(0|-1) \\ \text{-----} \\ \hat{\underline{b}}(0|-1) \end{bmatrix} \text{ and } P(0|-1) \quad (2.16)$$



where  $P(0|-1)$  is defined as the error covariance of the coefficients,

$$P(k|k-1) = E \left\{ \left[ \begin{array}{c} \underline{a}(k) \\ \text{---} \\ \underline{b}(k) \end{array} - \begin{array}{c} \hat{\underline{a}}(k|k) \\ \text{---} \\ \hat{\underline{b}}(k|k) \end{array} \right] \left[ \begin{array}{c} \underline{a}(k) \\ \text{---} \\ \underline{b}(k) \end{array} - \begin{array}{c} \hat{\underline{a}}(k|k) \\ \text{---} \\ \hat{\underline{b}}(k|k) \end{array} \right]^T \right\} \quad (2.17)$$

for  $k = 0$ . Equation (2.15b) is generally referred to as the Kalman gain. Two special cases are of interest: case (1);  $a_i = 0$  for all  $i$ , and case (2);  $b_i = 0$  for all  $i$ .

1. Autoregressive Form ( $a_i = 0$ )

For case 1, equation (2.7) takes the form,

$$y(k) = - \sum_{i=1}^n b_i y(k-i) + v(k). \quad (2.18)$$

This is a recursive equation stating that the present output is a linear combination of past outputs corrupted with additive gaussian noise,  $v(k)$ . Equation (2.18) is more formally recognized as an autoregressive process of order  $n$  [Ref. 25, 26]. Writing the Kalman solution for the coefficients of the AR process,

$$\hat{\underline{b}}(k+1|k) = [I - K(k)H(k)] \hat{\underline{b}}(k|k-1) + K(k)y(k) \quad (2.19a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.19b)$$





$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.19c)$$

Alternate forms of the Kalman equations given by Maybeck [Ref. 27] are,

$$\begin{aligned} \hat{\underline{b}}(k+1|k) &= P(k|k-1) P(k|k-1)^{-1} \hat{\underline{b}}(k|k-1) \\ &+ P(k+1|k) H^T(k) R^{-1} Y(k) \end{aligned} \quad (2.20a)$$

$$P(k+1|k) = (P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)^{-1}. \quad (2.20b)$$

We can model the fact that we have no a-priori knowledge about the initial values of the coefficients by letting,

$$[P(0|-1)]^{-1} \approx 0. \quad (2.21)$$

Further, if we remain ignorant and totally doubt our previous estimate, then  $P(k|k-1)$  is modeled as,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.22)$$

Equations (2.20) reduce to,

$$\hat{\underline{b}}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} Y(k), \quad (2.23)$$

the weighted least squares estimate, previously encountered by Swerling [Ref. 28,29,30], of the coefficients,  $\underline{b}$ . Carrying the analysis further and letting  $R^{-1} = 1$ , we have the Penrose pseudo-inverse solution [Ref. 31,32],

$$\hat{\underline{b}}(k+1|k) = [H(k) H^T(k)]^{-1} H(k) Y(k) \quad (2.24)$$



Swerling [Ref. 28] has shown that if the weighting matrix in (2.23) is not the inverse of the covariance matrix of the measurement errors, then the accuracy of the estimated coefficients (2.24) will be degraded.

The aforementioned notwithstanding, we press further into the analysis of equation (2.24). The product  $H(k)H^T(k)$  can be written as,

$$H(k)H^T(k) = \begin{bmatrix} y(k-1) \\ y(k-2) \\ \vdots \\ y(k-n) \end{bmatrix} [y(k-1) \ y(k-2) \ \dots \ y(k-n)] \quad (2.25)$$

$$H(k)H^T(k) = \begin{bmatrix} y^2(k-1) & y(k-1)y(k-2) & \dots & y(k-1)y(k-n) \\ y(k-2)y(k-1) & y^2(k-2) & & \vdots \\ \vdots & & & \vdots \\ y(k-n)y(k-1) & \dots & \dots & y(k-n)y(k-n) \end{bmatrix} \quad (2.26)$$

As we let  $k \rightarrow \infty$  the expected value of (2.26) becomes the autocorrelation matrix,

$$E\{\lim_{k \rightarrow \infty} H(k) H^T(k)\} = R_{YY}(k) \quad (2.27)$$

where,



$$R_{YY}(k) = \begin{bmatrix} R_{YY}(0) & R_{YY}(1) & \dots & R_{YY}(n) \\ R_{YY}(-1) & R_{YY}(0) & . & . \\ . & . & . & . \\ . & . & . & . \\ R_{YY}(-n) & \dots & \dots & R_{YY}(0) \end{bmatrix} . \quad (2.28)$$

Similarly, the product  $H(k)y(k)$  as  $k \rightarrow \infty$  becomes

$$r_{YY}(k) = \begin{bmatrix} R_{YY}(-1) \\ R_{YY}(-2) \\ . \\ . \\ . \\ R_{YY}(-n) \end{bmatrix} . \quad (2.29)$$

The steady state solution for the estimate of the coefficients  $\hat{\underline{b}}(k+1|k)$  is,

$$\hat{\underline{b}}_{ss}(k+1|k) = R_{YY}^{-1}(k) r_{YY}(k) . \quad (2.30)$$

Equation (2.30) is one of the starting points from which Perry [Ref. 6] develops his Lattice modeling algorithms. Equation (2.30) can also be recognized as the solution to the Yule-Walker or Normal equations [Ref. 26].

## 2. Moving Average Form ( $b_i = 0$ )

Referring once again to equation (2.7) and setting the coefficients  $b_i = 0$  for all  $i$  we have case (2),

$$y(k) = \sum_{i=0}^m u(k-i) + v(k) \quad (2.31)$$



Since  $v(k)$  is by definition noise associated with the measurement, we can combine  $y(k)$  and  $v(k)$  such that,

$$z(k) = y(k) - v(k) = \sum_{i=0}^m a_i u(k-i). \quad (2.32)$$

Equation (2.32) simply states that the present measurement is a linear combination of past and present inputs or by definition, a Moving Average (MA) process [Ref. 25,33]. The Adaptive Kalman Identifier estimate for the coefficients of the MA process is as before,

$$\hat{\underline{a}}(k+1|k) = [I - K(k)H(k)] \hat{\underline{a}}(k+1|k) + K(k)z(k) \quad (2.33a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.33b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.33c)$$

Using the alternate forms of the above equations we arrive at,

$$\begin{aligned} \hat{\underline{a}}(k+1|k) &= [P(k+1|k) [P(k|k-1)]^{-1}] \hat{\underline{a}}(k|k-1) \\ &\quad + [P(k+1|k) H^T(k) R^{-1}] z(k) \end{aligned} \quad (2.34a)$$

$$P(k+1|k) = [[P(k|k-1)]^{-1} + H^T(k) R^{-1} H(k)]^{-1}. \quad (2.34b)$$

Arguing as we did for the autoregressive case, no a-priori knowledge about the initial values of the moving average coefficients, implies that,





$$[P(0|-1)]^{-1} \approx 0. \quad (2.35)$$

And even if after we processed one measurement we still admitted no knowledge as to the accuracy of the previous estimate, we imply that,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.36)$$

Substituting these implications into equations (2.34) our estimate becomes,

$$\hat{\underline{a}}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} z(k) \quad (2.37)$$

the by now familiar weighted least squares estimate [Ref. 28, 29,30]. If once again we allow  $R^{-1}$  to be unity, let  $k$  tend toward infinity and take the expectation of equation (2.37), one arrives at the discrete form of the Wiener-Hopf equation [Ref. 11], namely

$$\hat{\underline{a}}(k+1|k) = R_{uu}^{-1}(k) r_{uz}(k) \quad (2.38)$$

where it can easily be shown that,

$$R_{uu}(k) = \begin{bmatrix} R_{uu}(0) & R_{uu}(-1) & \dots & R_{uu}(-m) \\ R_{uu}(-1) & R_{uu}(0) & \dots & R_{uu}(1-m) \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ R_{uu}(-m) & \dots & & R_{uu}(0) \end{bmatrix} \quad (2.39)$$

and,



$$R_{uy}(k) = \begin{bmatrix} R_{uy}(0) \\ R_{uy}(-1) \\ \vdots \\ R_{uy}(-m) \end{bmatrix} \quad (2.40)$$

Equation (2.38) provided another point of departure from which Perry [Ref. 6] develops the MA Lattice modeling algorithms. Appendix A develops the Wiener-Hopf equation otherwise known as the all-zero model from yet another approach.

The all-zero model is fundamental to Widrow least mean square (LMS) adaptive filters [Ref. 11] and linear prediction theory [Ref. 16].

### 3. Autoregressive-Moving Average Form

Returning to the alternate form (equations 2.20) of the Kalman Filter equations (2.19) the development of the Autoregressive Moving Average (ARMA) Adaptive Kalman Identifier follows. The estimate for the  $a_i$  and  $b_i$  coefficients is,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \text{-----} \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} = [P(k+1|k) [P(k|k-1)]^{-1}] \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \text{-----} \\ \hat{\underline{b}}(k|k-1) \end{bmatrix} + [P(k+1|k) H^T(k) R^{-1}] y(k) \quad (2.41a)$$

$$P(k+1|k) = [(P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)]^{-1} \quad (2.41b)$$



Progressing in the same manner as in the previous two cases,  
we assume,

$$[P(0|-1)]^{-1} \approx 0 \quad (2.42a)$$

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.42b)$$

The estimate then becomes,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = [H^T(k) R^{-1} H(k)]^{-1} H(k) R^{-1} y(k). \quad (2.43)$$

Taking equation (2.43) a step further by letting  $R^{-1}$  be  
unity, letting  $k$  approach infinity and taking the expectation  
we have,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{nu}(k) & -R_{uy}(k-1) \\ \hline -R_{uy}^T(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ \hline R_{yy}(k) \end{bmatrix} \quad (2.44)$$

Note that the time varying measurement vector,  $H(k)$ , is of  
the form

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-n) \mid -y(k-1) \ -y(k-2) \ \dots \ -y(k-n)] \quad (2.45)$$

Shown in detail, equation (2.44) has the characteristic form  
(equation 2.46, next page). It is the Toeplitz and symmetric  
nature of equation (2.46) that is exploited by the Levinson  
[Ref. 34] and Lattice [Ref. 35] algorithms.



$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix}$$

=

$$\begin{bmatrix} R_{uu}(0) & R_{uu}(-1) & \dots & R_{uu}(-m) & -R_{uy}(-1) & -R_{uy}(-2) & \dots & -R_{uy}(-n) \\ R_{uu}(-1) & R_{uu}(0) & \dots & R_{uu}(1-m) & -R_{uy}(0) & -R_{uy}(-1) & \dots & -R_{uy}(1-n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{uu}(1-m) & \dots & R_{uu}(0) & -R_{uy}(m-1) & \dots & \dots & -R_{uy}(m-n) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(0) \\ R_{uy}(1) \\ \vdots \\ R_{uy}(m) \\ R_{yy}(1) \\ R_{yy}(2) \\ \vdots \\ R_{yy}(n) \end{bmatrix}$$

$$\begin{bmatrix} -R_{yu}(-1) & -R_{yu}(0) & \dots & -R_{yu}(m-1) & R_{yy}(0) & R_{yy}(-1) & \dots & R_{yy}(-n) \\ -R_{yu}(-2) & -R_{yu}(-1) & \dots & \cdot & R_{yy}(-1) & R_{yy}(0) & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_{yu}(-n) & \dots & -R_{yu}(m-n) & R_{yy}(-n) & \dots & \dots & R_{yy}(0) \end{bmatrix}$$

(2.46)





The Adaptive Kalman ARMA modeling technique can be best visualized by a block diagram. Referring to Figure 2.1, the unknown system is excited by a white, zero mean, gaussian noise sequence of sample values from a random process. The input and its associated output are then passed through M and N delays respectively in serial form. The parallel inputs (M+1) and the outputs (N) are concatenated to form the measurement vector  $H(k)$  which is represented by equation (2.13). The inputs N and M are selected a-priori as the model orders for the Autoregressive and Moving Average processes one desires to identify. It can be easily seen that the remainder of the figure simply implements equation (2.15a). The outputs of the Adaptive Kalman Identifier are an estimate of the coefficients,

$$\begin{bmatrix} \hat{\underline{b}}(k|k) \\ \text{-----} \\ \hat{\underline{a}}(k|k) \end{bmatrix}$$

and a one step prediction,  $\hat{y}(k|k)$ .

### C. OBSERVATIONS

In this section it has been shown that a direct connection can be established between the Adaptive Kalman Identifier and the Yule-Walker equations associated with an AR process. Secondly, the steady state Adaptive Kalman Identifier closely resembles the discrete Weiner-Hopf equation associated with the MA process when the measurement matrix,  $H(k)$ , is of the form,



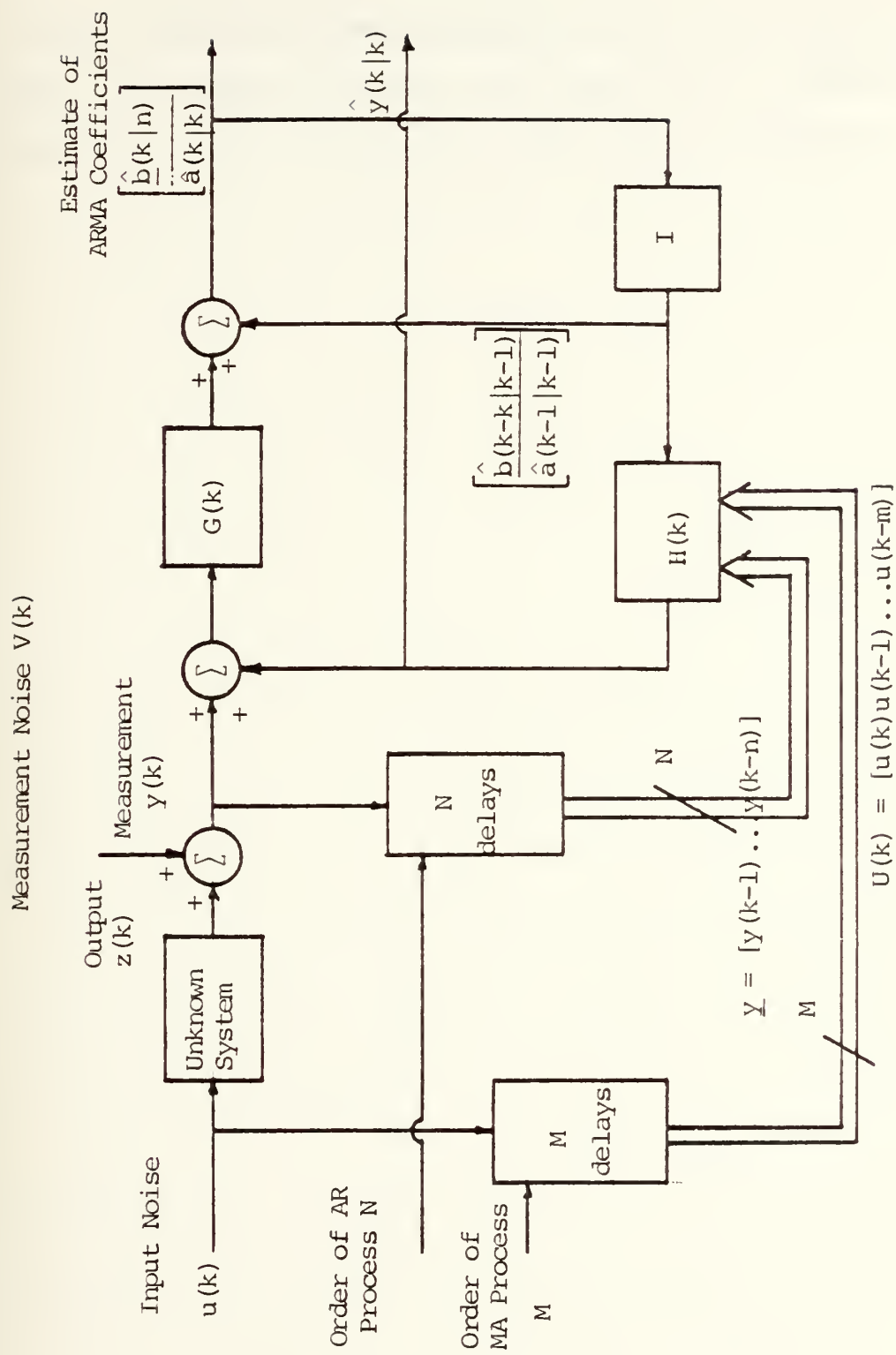


Figure 2.1. Adaptive Kalman Identifier



$$H(k) = [u(k) \ u(k-1) \ u(k-2) \ \dots \ u(k-m)]$$

And, thirdly under the same assumptions as in the previous two cases, the Adaptive Kalman ARMA Identifier is similar to the form Perry [Ref. 6] exploits using Lattice modeling.



### III. COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND THE WIDROW LMS ADAPTIVE FILTER

#### A. PRELIMINARIES

It is instructive to investigate the similarities between the Adaptive Kalman Identifier and the Widrow LMS Adaptive Filter when the concepts are applied to system identification. Basically, the LMS algorithm implementation of system identification considers a block diagram as is shown in Figure 3.1. The output,  $y(k)$ , of the Adaptive filter is simply a weighted linear combination of the past and present known inputs. The same input is fed into both the unknown system to be identified and the adaptive filter. The output of the unknown system is designated the desired response,  $d(k)$ , from which an error signal,  $\epsilon(k)$ , is derived. The error signal,  $\epsilon(k)$ , provides the criterion through which the weights,  $w_i$ , are adjusted such that the error,  $\epsilon(k+1)$ , is driven toward its minimum. A more detailed analysis of the operation of the LMS algorithm can be found in Ref. 11.

The weights  $w_i$  are adjusted from time step to time step in the following manner,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \epsilon(k) \underline{X}(k) \quad (3.1)$$

where we define the following quantities,





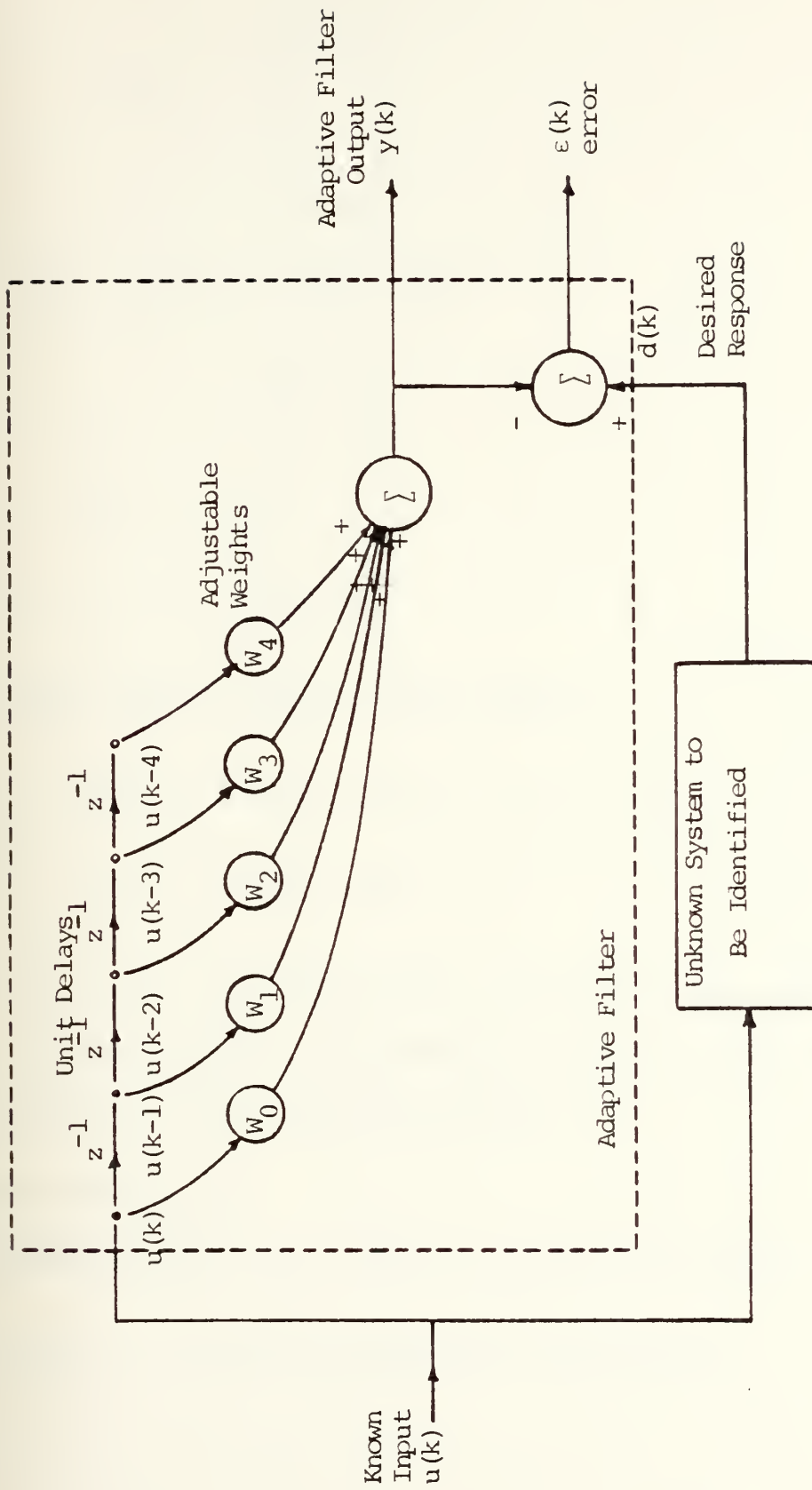


Figure 3.1. LMS Adaptive Filter Implementation of System Identification



$$\underline{w}(k) = \begin{bmatrix} w_0(k) \\ w_1(k) \\ \vdots \\ w_m(k) \end{bmatrix} \quad (3.2)$$

the weight vector at time step  $k$ ,

$$\underline{X}(k) = \begin{bmatrix} u(k) \\ u(k-1) \\ \vdots \\ u(k-m) \end{bmatrix} \quad (3.3)$$

the input signal vector at time step  $k$ ,

$$\epsilon(k) = d(k) - y(k) \quad (3.4)$$

$$y(k) = \underline{w}^T(k) \underline{X}(k) \quad (3.5)$$

the error,  $\epsilon(k)$ , and the output,  $y(k)$ , at time step  $k$ , and,  $k_s$ , a scalar constant controlling the rate of convergence and the stability of the adaptive filter.

#### B. THE COMPARISON

Substituting (3.4) and (3.5) into (3.1) gives,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \underline{X}(k) [d(k) - \underline{w}^T(k) \underline{X}(k)]. \quad (3.6)$$



Recalling the MA form of the Adaptive Kalman Identifier, equation (2.33a), rewritten here for convenience as

$$\hat{\underline{a}}(k+1|k) = \hat{\underline{a}}(k|k-1) + K(k) [z(k) - H(k) \hat{\underline{a}}(k|k-1)], \quad (3.7)$$

one can make the following associations:

$$\hat{\underline{a}}(k+1|k) \langle \Rightarrow \underline{w}(k+1) \quad (3.8a)$$

$$\hat{\underline{a}}(k|k-1) \langle \Rightarrow \underline{w}(k) \quad (3.8b)$$

$$z(k) \langle \Rightarrow d(k) \quad (3.8c)$$

$$H^T(k) \langle \Rightarrow \underline{x}(k) \quad (3.8d)$$

$$K(k) \langle \Rightarrow -2k_s \underline{x}(j) \quad (3.8e)$$

Recall that for the MA form of the AKI, the measurement vector  $H(k)$  represents a vector of past and present inputs. Namely,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m)] . \quad (3.9)$$

Therefore the associations (3.8a)-(3.8d) are straightforward. However, it is not so clear as to what is meant by the association (3.8e). Digressing a moment to present an equivalent expression [Ref. 27] for the Kalman gain,  $K(k)$ ,

$$K(k) = P(k|k) H^T(k) R^{-1} \quad (3.10)$$

and substituting (3.10) into the association (3.8e) and entertaining the conjecture that the quantities are equivalent



under certain conditions, we have:

$$P(k|k) H^T(k) R^{-1} = -2k_s \underline{X}(k) . \quad (3.11)$$

The conditions alluded to are (1) that the Adaptive Kalman Identifier is in steady state and (2) that the statistics of the input forcing function are stationary. Denoting the steady state error covariance,  $P(k|k)$ , as  $P_\infty$ , equation (3.11) can be solved for  $k_s$ ,

$$k_s I = -\frac{1}{2} R^{-1} P_\infty . \quad (3.12)$$

Invoking the entire Kalman gain history in its more popular form of equation (2.33b) and equating it to the Widrow gain (3.8e), we instead arrive at,

$$-2k_s \underline{X}(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (3.13a)$$

$$H^T(k) = \underline{X}(k) . \quad (3.13b)$$

Solving for the convergence factor,  $k_s$ , we obtain,

$$-2k_s \underline{X}(k) \underline{X}^T(k) = P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k)$$

$$-2k_s I = P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k) [\underline{X}(k) \underline{X}^T(k)]^{-1} .$$

But  $[\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1}$  is a scalar. Therefore,

$$-2k_s I = \frac{P(k|k-1)}{\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R} \quad (3.14)$$





Griffiths [Ref. 22] suggests that the convergence factor  $k_s$ , which he denotes as  $\mu(n)$ , should be chosen such that,

$$\mu(n) = \frac{\alpha}{L \hat{\sigma}_x^2(n)} \quad (3.15)$$

where  $0 < \alpha < 1$  is a normalized adaptive stepsize parameter and the term  $\hat{\sigma}_x^2(n)$  is an estimate of the input power level which may be computed using a geometrically-weighted average for an  $L$  weight adaptive filter,

$$\hat{\sigma}_x^2(n) = (1 - \frac{\alpha}{L}) \hat{\sigma}_x^2(n-1) + \frac{\alpha}{L} x^2(n) . \quad (3.16)$$

Comparing equation (3.14) and (3.15), one observes that if  $P(k, k-1)$  is equal to the identity matrix then we obtain a term proportional to the input power. Further if measurement noise is considered negligible,  $R \rightarrow 0.0$ , then 3.14 and 3.15 are essentially equivalent. The salient point to make, however, is that the choice of the convergence constant,  $k_s$ , in the LMS algorithm provides no clue as to how to deal with measurement noise whereas the Adaptive MA Kalman Identifier explicitly incorporates measurement error into its algorithm.

### C. OBSERVATIONS

It has been known that the LMS algorithm was suboptimal since the actual gradient of equation (3.1) is replaced by the approximation,  $2\varepsilon(k) x(k)$ , [Ref. 24]. However, the degree of suboptimality is difficult to quantify. In previous linear prediction research no mention was made regarding the



role of measurement noise. Equation (3.14) gives us some insight into wiser selections of the rate of convergence constant,  $k_s$ , since it takes into account the effects of measurement noise. Further, it appears that the LMS adaptive filter is a special case of the Adaptive MA Kalman Identifier.



#### IV. COMPARISON BETWEEN THE ADAPTIVE ARMA KALMAN IDENTIFIER AND THE ADAPTIVE RECURSIVE LMS FILTER

This chapter endeavors to investigate the similarities and differences between the Adaptive ARMA Kalman Identifier and the Adaptive Recursive LMS Filter [Ref. 18]. LMS filters [Ref. 11] have enjoyed much popularity in the recent past due to their ease of implementation, simple unimodal algorithm, robustness and ability to "adapt" to the unknown statistics of the signal environment. Being transversal in nature, they have a finite impulse response being able to produce only zeros in the input/output transfer function. One may, however, decide to model the transfer function,

$$H_1(z) = \frac{1}{1 - .9z^{-1}} \quad (4.1)$$

using a transversal filter, only to realize that a large number of delays are required in order to arrive at an adequate approximation. The germane point which is being made is that one weighted feedback tap can realize an infinite string of feed-forward coefficients. Moreover, it is very desirable to adjust the feedback weights adaptively in some optimal fashion to the statistics of the signal environment.

##### A. PRELIMINARIES

The approach taken by Feintuch [Ref. 18] is patterned after the analysis first presented by Widrow [Ref. 11]. A summary of Feintuch's derivation will be presented here with emphasis on its application to ARMA modeling.



Recall the general ARMA equation (2.1b),

$$y(k) = \sum_{i=0}^m a_i u(k-i) + \sum_{i=1}^n b_i y(k-i) \quad (4.2a)$$

which can be rewritten in vector notation,

$$y(k) = \underline{a}^T \underline{u} + \underline{b}^T \underline{y} \quad (4.2b)$$

where,

$$\underline{a}^T = [a_0 \ a_1 \ \dots \ a_m] \quad (4.3a)$$

$$\underline{b}^T = [b_1 \ b_2 \ \dots \ b_n] \quad (4.3b)$$

$$\underline{y} = [y(k-1) \ y(k-2) \ \dots \ y(k-n)]^T \quad (4.3c)$$

$$\underline{u} = [u(k) \ u(k-1) \ \dots \ u(k-m)]^T \quad (4.3d)$$

Given that (4.2) is the assumed mathematical description of the unknown system where  $\underline{u}$  and  $\underline{y}$ , the input and output data sequences, are known, then by solving (4.2) for  $\underline{a}$  and  $\underline{b}$ , identification of the unknown system can be made. The LMS algorithms, in general, employ a "desired" signal  $d(k)$  with which to "train" the adaptive filter. If the desired signal was assumed to be the response of some unknown system to a known input signal, then the algorithm presented by Feintuch [Ref. 18] can be used as a means by which to identify the system parameters.

The first step in the Recursive LMS derivation is to form the error between the desired signal,  $d(k)$ , and the output





of the filter,  $y(k)$ ,

$$e(k) = d(k) - y(k) = d(k) - \underline{a}^T \underline{u} - \underline{b}^T \underline{y} \quad (4.4)$$

Forming the square of (4.4) and taking the expectation, we have the mean square error representation of the filter,

$$\begin{aligned} E\{e^2(k)\} &= E\{d^2(k)\} + \underline{a}^T R_{uu} \underline{a} + \underline{b}^T R_{yy} \underline{b} \dots \\ &\quad - 2\underline{a}^T R_{du} - 2\underline{b}^T R_{dy} + 2\underline{a}^T R_{uy} \underline{b} \end{aligned} \quad (4.5)$$

where,

$$R_{uu}(k) = E\{u(k) u^T(k)\} \quad (4.6a)$$

$$R_{yy}(k-1) = E\{y(k-1) y^T(k-1)\} \quad (4.6b)$$

$$R_{du}(k) = E\{d(k) u(k)\} \quad (4.6c)$$

$$R_{dy}(k) = E\{d(k) y(k)\} \quad (4.6d)$$

$$R_{uy}(k-1) = E\{u(k-1) y(k-1)\} . \quad (4.6e)$$

It is assumed that the statistics (4.6) are constants allowing the gradient of (4.5) to be taken with respect to the  $a_i$  and  $b_i$ . This assumption does not stretch the theory since in practice one uses an input test signal with stationary characteristics and if the unknown system's output process is not stationary, then no identification of the system parameters can be made. The respective gradients are,



$$\frac{\partial}{\partial \underline{a}}[E\{e^2(k)\}] = 2R_{uu}(k)\underline{a} - 2R_{du}(k) + 2R_{uy}(k)\underline{b} \quad (4.7a)$$

$$\frac{\partial}{\partial \underline{b}}[E\{e^2(k)\}] = 2R_{yy}(k)\underline{b} - 2R_{dy}(k) + 2R_{uy}^T(k)\underline{a} . \quad (4.7b)$$

Since the second order statistics are assumed to be known, equations (4.7) can be solved for  $\underline{a}$  and  $\underline{b}$  by setting the gradients equal to zero. In matrix partitioned form we have,

$$\begin{bmatrix} R_{uu}(k) & R_{uy}(k) \\ R_{uy}^T(k) & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ R_{dy}(k) \end{bmatrix} . \quad (4.8)$$

At this point we digress momentarily to mention that Johnson and Larimore [Ref. 19] and Widrow and McCool [Ref. 20] agree with Feintuch's derivation. The following steps, however, are controversial [Ref. 19,20].

Feintuch continues and states that in general, the statistics involved in (4.8) are not available a-priori. One method of estimating the statistics is to make the filter adaptive in an iterative fashion using the method of steepest descent. The method of steepest descent employs an algorithm of the form,

$$\underline{a}(k+1) = \underline{a}(k) + k_a \frac{\partial}{\partial \underline{a}}[E\{e^2(k)\}] \quad (4.9a)$$

$$\underline{b}(k+1) = \underline{b}(k) + k_b \frac{\partial}{\partial \underline{b}}[E\{e^2(k)\}]. \quad (4.9b)$$

The gradient involved in (4.9) can be approximated using the techniques outlined in [Ref. 11]. The iterative LMS algorithm



for estimating the coefficients  $\underline{a}$  and  $\underline{b}$  is,

$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.10a)$$

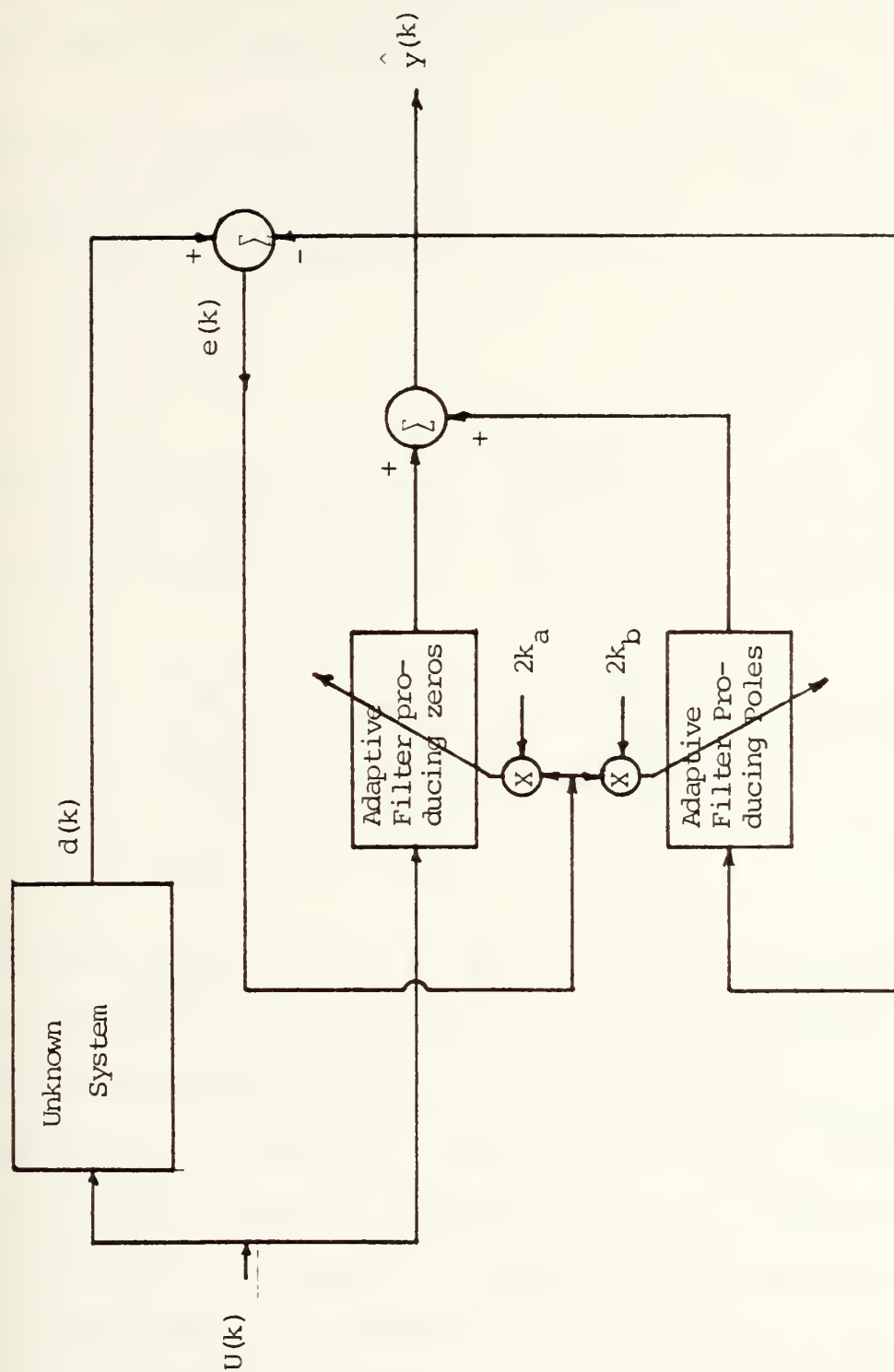
$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \underline{y}(k). \quad (4.10b)$$

Figure 4.1 is a block diagram implementation of the Adaptive Recursive LMS Filter algorithm as applied to system identification. The input signal,  $u(k)$ , to both the unknown system and the adaptive filter is a zero mean, white gaussian noise sequence of samples from a random process. The output of the unknown system is  $d(k)$  which is used as the training signal for the filter. The output of the adaptive filter,  $y(k)$ , is compared with the desired signal,  $d(k)$ , from which an error signal is derived. The error signal,  $e(k)$ , is then used in the LMS algorithm, equation (4.10), to iteratively adjust the  $\underline{a}$  and  $\underline{b}$  coefficients. Theoretically, when the coefficients of the zeros-producing adaptive filter and the poles-producing adaptive filter correspond exactly with those of the unknown system, the error sequence should be zero and the unknown system is modeled by the Adaptive Recursive LMS Filter. The convergence constants,  $k_a$  and  $k_b$ , are chosen a-priori as are the initial values,  $\underline{a}(0)$  and  $\underline{b}(0)$ , for the coefficients.

## B. THE COMPARISON

One can begin the comparison between the Adaptive Kalman Identifier and the Adaptive Recursive LMS filter by noting





Filter 4.1. Adaptive Recursive LMS Filter Applied to System Identification





the similarities between the equations (2.43) and (4.8), repeated here for convenience,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & -R_{uy}(k-1) \\ \hline -R_{yu}^T(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ \hline R_{yy}(k) \end{bmatrix} \quad (2.43a)$$

$$A_1 = B_1 C_1 \quad (2.43b)$$

$$\begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ \hline R_{uy}^T(k-1) & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ \hline R_{dy}(k) \end{bmatrix} \quad (4.8a)$$

$$\begin{bmatrix} \hat{\underline{a}} \\ \hline \hat{\underline{b}} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ \hline R_{uy}^T(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ \hline R_{dy}(k) \end{bmatrix} \quad (4.8b)$$

$$A_2 = B_2 C_2 \quad (4.8c)$$

The upper left partitions of  $B_1$  and  $B_2$  are obviously identical, in that the AKI and the Adaptive Recursive LMS both implement instantaneous estimates of the input covariance functions,  $R_{uu}(k)$ . The lower right partitions of  $B_1$  and  $B_2$  are similar with one subtle difference. Both are instantaneous covariance functions; however,  $B_1$  employs the covariance,  $R_{yy}(k)$ , of the actual data whereas  $B_2$  uses, in fact, previous filter estimates to compute the output covariance,  $R_{yy}(k)$ , more properly denoted by  $\hat{\hat{R}}_{yy}(k)$ . The negative sign of the upper right and lower left partitions in  $B_1$ , in comparison with  $B_2$ , is a result of the initial definition for the recursive



weights,  $\underline{b}_i$ . That is, the transfer function for the general ARMA equation can be written as,

$$H_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 - \sum_{i=1}^m b_i z^{-i}} \quad (4.9a)$$

or,

$$H'_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 + \sum_{i=1}^n b_i z^{-i}} \quad (4.9b)$$

The second and more important observation is that cross-correlations employed by equation (4.8b) use the instantaneous values for the past estimates of the outputs while (2.43a) uses the actual past values for  $y(k)$ . Similarly, comparing  $C_1$  with  $C_2$ , one finds that the lower partition of  $C_2$  uses the cross-correlation between the desired signal and the past estimates of the adaptive recursive LMS filter.

Equation (4.8a) can convey the above information more clearly if it is instead written as,

$$\begin{bmatrix} \hat{\underline{a}} \\ \hat{\underline{b}} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & R_{u\hat{y}}(k-1) \\ R_{u\hat{y}}^T(k-1) & R_{\hat{y}\hat{y}}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ R_{d\hat{y}}(k) \end{bmatrix} \quad (4.10)$$

The convergence factors,  $k_a$  and  $k_b$ , can be compared to the steady state Adaptive Kalman Identifier gains by making



the identical analogies (3.8) as was done in the previous chapter. One also observes that since the output data that is processed by the Adaptive Recursive LMS filter are in actuality filter estimates, a more correct version of Feintuch's original algorithm (4.10) is written as,

$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.11a)$$

$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \hat{\underline{y}}(k) . \quad (4.11b)$$

In augmented form, equation (4.11) is,

$$\begin{bmatrix} \underline{a}(k+1) \\ \underline{b}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{a}(k) \\ \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{u}(k) \\ 2k_b \underline{y}(k) \end{bmatrix} [d(k) - \underline{a}(k) \underline{u}(k) - \underline{b}(k) \underline{y}(k)]$$

$$\begin{bmatrix} \underline{a}(k+1) \\ \underline{b}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{a}(k) \\ \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{u}(k) \\ 2k_b \underline{y}(k) \end{bmatrix} \begin{bmatrix} d(k) - [\underline{u}(k) \quad \underline{y}(k)] \begin{bmatrix} \underline{a}(k) \\ \underline{b}(k) \end{bmatrix} \end{bmatrix} . \quad (4.12)$$

Recalling equation (2.15a) in a slightly different form we have,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k) \begin{bmatrix} z(k) - H(k) \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hat{\underline{b}}(k|k-1) \end{bmatrix} \end{bmatrix} . \quad (4.13)$$

It is a simple matter to explore the similarities between (4.12) and (4.13) and make the following associations,



$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \text{-----} \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} \quad \langle \Rightarrow \quad \begin{bmatrix} \underline{a}(k+1) \\ \text{-----} \\ \underline{b}(k+1) \end{bmatrix} \quad (4.14a)$$

$$\begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \text{-----} \\ \hat{\underline{b}}(k|k-1) \end{bmatrix} \quad \langle \Rightarrow \quad \begin{bmatrix} \underline{a}(k) \\ \text{-----} \\ \underline{b}(k) \end{bmatrix} \quad (4.14b)$$

$$z(k) \quad \langle \Rightarrow \quad d(k) \quad (4.14c)$$

$$H(k) = \begin{bmatrix} u(k) & \dots & u(k-m) & y(k-1) & \dots & y(k-n) \end{bmatrix} \quad \langle \Rightarrow \quad [\underline{u}(k) \quad \underline{y}(k)] \quad (4.14d)$$

$$K(k) \quad \langle \Rightarrow \quad \begin{bmatrix} 2k_a \underline{u}(k) \\ \text{-----} \\ 2k_b \underline{y}(k) \end{bmatrix} \quad (4.14e)$$

The associations (4.14d) and (4.14e) are the major differences between the AKI and the Adaptive Recursive LMS. It has been shown that the mean square error surface employing not only estimated feed forward coefficients but also estimated feed-back (recursive) coefficients is in general multimodal [Ref. 36]. Hence, the Adaptive Recursive LMS algorithm does not minimize the mean square error [Ref. 19,20], and in general the gradient algorithm in this case does not seek the global minimum.





### C. OBSERVATIONS

The complex nature of estimating recursive coefficients is yet a formidable problem [Ref. 19,20]. It seems that Feintuch's algorithm is successful due to the a-priori knowledge of the minimal order generating the desired signal. This knowledge is used in setting the order of the adaptive recursive filter.



## V. SYNTHETIC DATA GENERATING PLANTS

The Adaptive ARMA Kalman Identifier was tested using computer derived data from several models. The input/output data was collected by driving a known plant with a zero mean, unit variance, white gaussian noise sequence of samples. The AKI algorithm was repeatedly tested using data from progressively more complex models.

### A. AUTOREGRESSIVE-MOVING AVERAGE DATA

Autoregressive-Moving Average (ARMA) data was generated using an equation of the form,

$$\begin{aligned} y(k) = & b_1 y(k-1) + b_2 y(k-2) + \dots + b_n y(k-n) \\ & + a_0 u(k) + \dots + a_m u(k-m) \end{aligned} \quad (5.1)$$

Taking the Z-transform of (5.1) we have,

$$\begin{aligned} Y(z) = & b_1 z^{-1} Y(z) + b_2 z^{-2} Y(z) + \dots + b_n z^{-n} Y(z) \\ & + a_0 U(z) + \dots + a_m z^{-m} U(z) \end{aligned} \quad (5.2)$$

where  $Y(z)$  ( $U(z)$ ) represents the Z-transform of the output (input) and  $z^{-n_0} Y(z)$  ( $z^{-n_0} U(z)$ ) represents the Z-transform of the output (input) delayed  $n_0$  time steps. Equation (5.1) has already been defined as the general form of the ARMA process provided that the sequence  $u(k-i)$   $i = 0, 1, \dots$  comes from a gaussian random process [Ref. 25]. The ratio,



$$H(z) = \frac{Y(z)}{U(z)} = \frac{a_0 + a_1 z^{-1} + \dots + a_m z^{-m}}{1 - b_1 z^{-1} - \dots - b_n z^{-n}} \quad (5.3)$$

can then be readily recognized as the general transfer function [Ref. 37] of a digital plant. That is, the  $u(k)$  is the observed input and the  $y(k)$  is the perfectly measured output. Consider that the measurements of the output include some random error,  $v(k)$ , due to the inaccuracy of the measuring instruments, then the resulting situation is as depicted in Figure 5.1. A practical, judicious and reasonable description of the measurement error is that this be a stochastic process whose distribution is zero mean gaussian. The noisy measurement,  $z(k)$ , is then the output,  $y(k)$ , plus the measurement error,  $v(k)$ .

There are three cases of interest: Case 1:  $b_i = 0$  for all  $i$ ; Case 2:  $a_i = 0$ ,  $i = 1, 2, \dots, m$ ; Case 3:  $a_i \neq 0$ ,  $b_i \neq 0$  for all  $i$ .

#### 1. Moving Average Data (Case 1)

Case one is readily recognized as the all zero plant which produces moving average data. A simple second order plant was defined where  $a_0 = 1.0$ ,  $a_1 = 2.0$  and  $a_2 = 3.0$ . The gaussian noise sequences,  $u(k)$  and  $v(k)$ , were obtained using the general purpose IMSL subroutine, GGNML, provided by the computer center at the Naval Postgraduate School.

#### 2. Autoregressive Data (Case 2)

Case two results in the all pole plant producing autoregressive data. The coefficients were selected such that the



plant of Figure 5.1 produced stable data. In precise terms, the poles of the function,

$$H(z) = \frac{1}{1 + \frac{26}{24} z^{-1} + \frac{9}{24} z^{-2} + \frac{1}{24} z^{-3}} \quad (5.4)$$

were located completely within the unit circle in the z-plane ensuring a stable plant [Ref. 37,38]. The noise sequences were obtained as before.

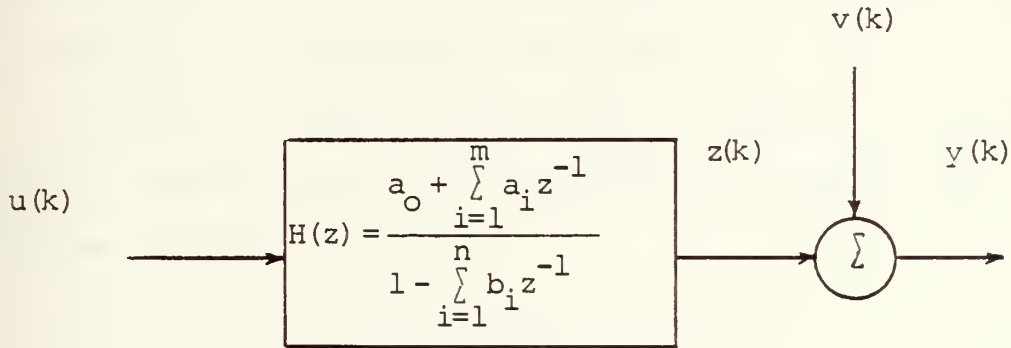


Figure 5.1. ARMA Digital Plant

### 3. Autoregressive-Moving Average Data (Case 3)

Case three follows from a logical combination of the two previous cases. That is, the transfer function,  $H(z)$ , now has both zeros and poles. In order to compare with previous work done in the area of system identification [Ref. 6], one of Perry's models [Ref. 6] was used. Namely, the transfer function  $H(z)$  used for this case was,

$$H(z) = \frac{1 + 1.4z^{-1} + .98z^{-2}}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \quad (5.5)$$





The input,  $u(k)$ , was obtained as in the previous two cases using the IMSL subroutine GGNML set to unit variance and zero mean. The output measurement noise sequence, however, was set to a variance of .0001. The basic software, program ARMA, which generated the different data is included as Appendix B. Basically, the coefficients A1-A10 change the character of the general ARMA equation (5.1) which can be selected to produce the desired plant data. The input/output data is then written onto a disk file for subsequent analysis.

In order to compare some of the findings in this thesis with the results previously obtained by Feintuch [Ref. 18], the transfer function,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1.0 - 1.1314z^{-1} + 0.25z^{-2}} \quad (5.6)$$

was also used as a source of synthetic data. Feintuch used (5.6) as a source of data with which to analyze the operation of his Adaptive Recursive LMS Filter.

## B. PHASE LOCKED LOOP DATA

The second class of data used to exercise the Adaptive ARMA Kalman Identifier was derived from a computer simulated phase locked loop (PLL) developed by Romeo [Ref. 7]. The basic PLL algorithm, however, implemented a forward Euler integration scheme.

Briefly, in block diagram form, the phase tracking characteristics of the PLL in the frequency domain can be depicted as in Figure 5.2.



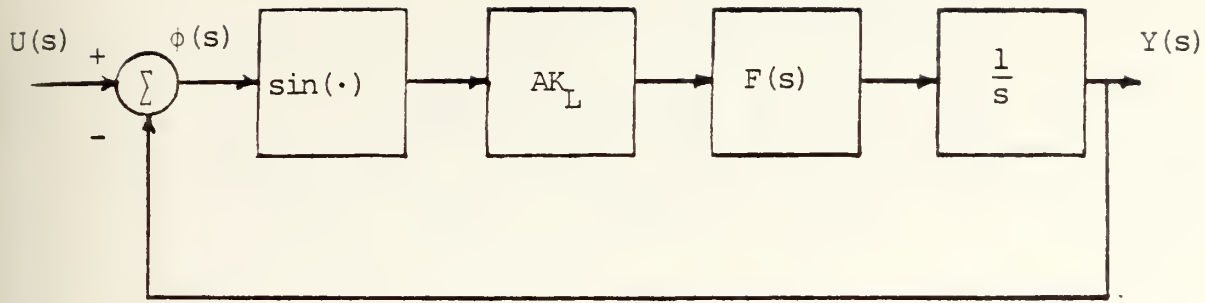


Figure 5.2. PLL Block Diagram

Where:

A is the input voltage

K is the loop gain

$F(s)$  is the filter transfer function

For most applications, it is assumed that  $\phi(t) \ll \pi/2$  allowing one to make the approximation,  $\sin \alpha \approx \alpha$ . This is the linear mode of operation. Romeo [Ref. 7] chose the filter characteristics  $F(s) = 1 + K/S$  and the same characteristics were used in this thesis in order to compare results. The parameters of the overall system were adjusted to obtain a damping coefficient,  $\zeta$ , of 0.3 and a natural frequency,  $\omega_n$  of 3.33 rad/sec. This resulted in a step response overshoot of about 46.7% at  $t \approx .75$  sec. Solving for the time domain step response analytically in the linear region one obtains,

$$y(t) = 1 + 1.048e^{-t} \sin(3.178t - 1.266) \quad (5.7)$$

The final block diagram of the PLL system is shown in Figure 5.3.



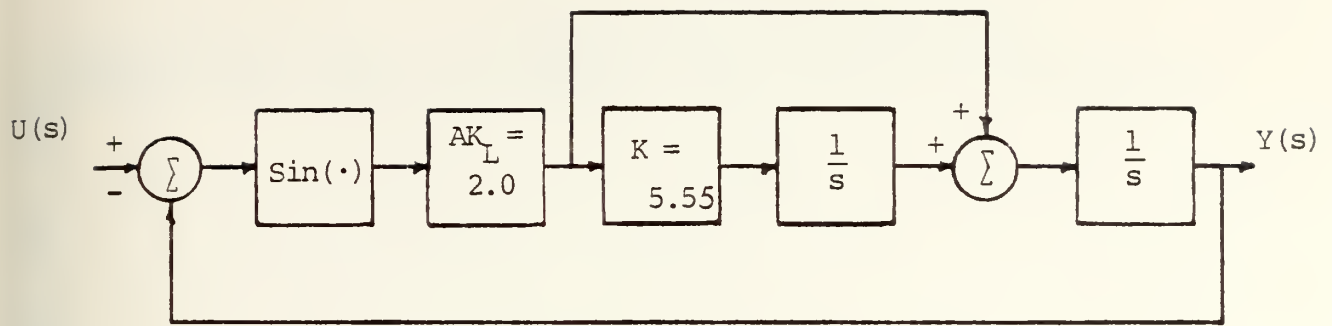


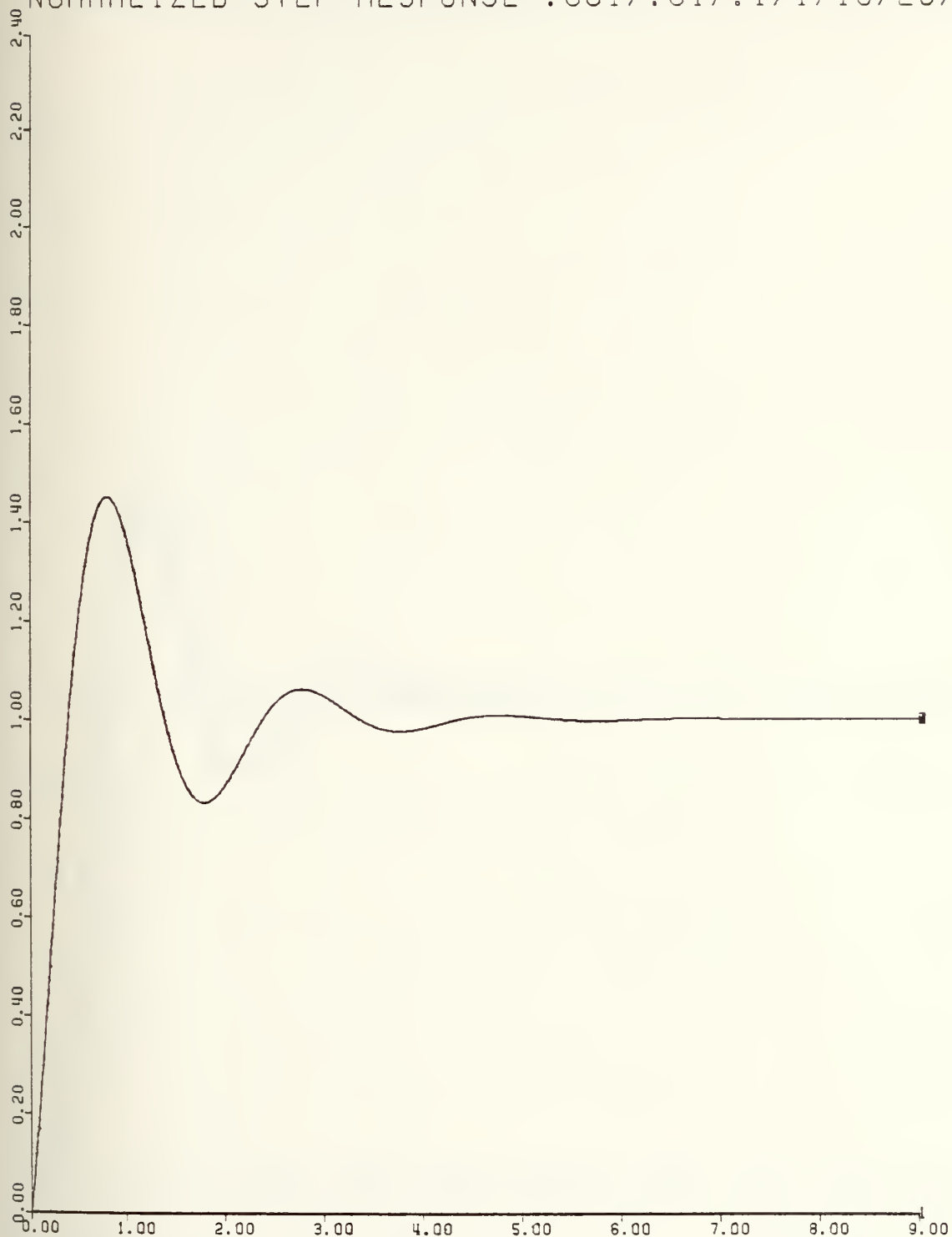
Figure 5.3. Final PLL Block Diagram

Normalized step responses for several step magnitudes using Digital Simulation Language (DSL) were obtained. Figure 5.4 shows the step responses of several step inputs in the range  $0 < u(t) < 30$ . It is obvious that the PLL is in its approximate linear region and does not exhibit any discernible distortion. Figure 5.5 is a repeat of the above test; however, the range of step inputs are  $30 \leq u(t) \leq 170$ . It is apparent from Figure 5.5 that the  $\sin(\cdot)$  nonlinearity begins to have some effect on the operation of the PLL in that the amplitude of the responses are reduced and delayed.

The DSL PLL system, however, was not used as a source of synthetic data because of the problem of nonstationary statistics of the input signal using the random sequence generators available under DSL already discussed in Ref. 7. These simulations were nevertheless used as a basis for comparison of the operation of the discrete PLL developed in Reference 7 but modified to implement a forward Euler integration scheme which is used in this thesis.



2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION  
NORMALIZED STEP RESPONSE .001/.01/.1/1/10/20/30



XSCALE= 1.00 UNITS/INCH  
YSCALE= 0.20 UNITS/INCH

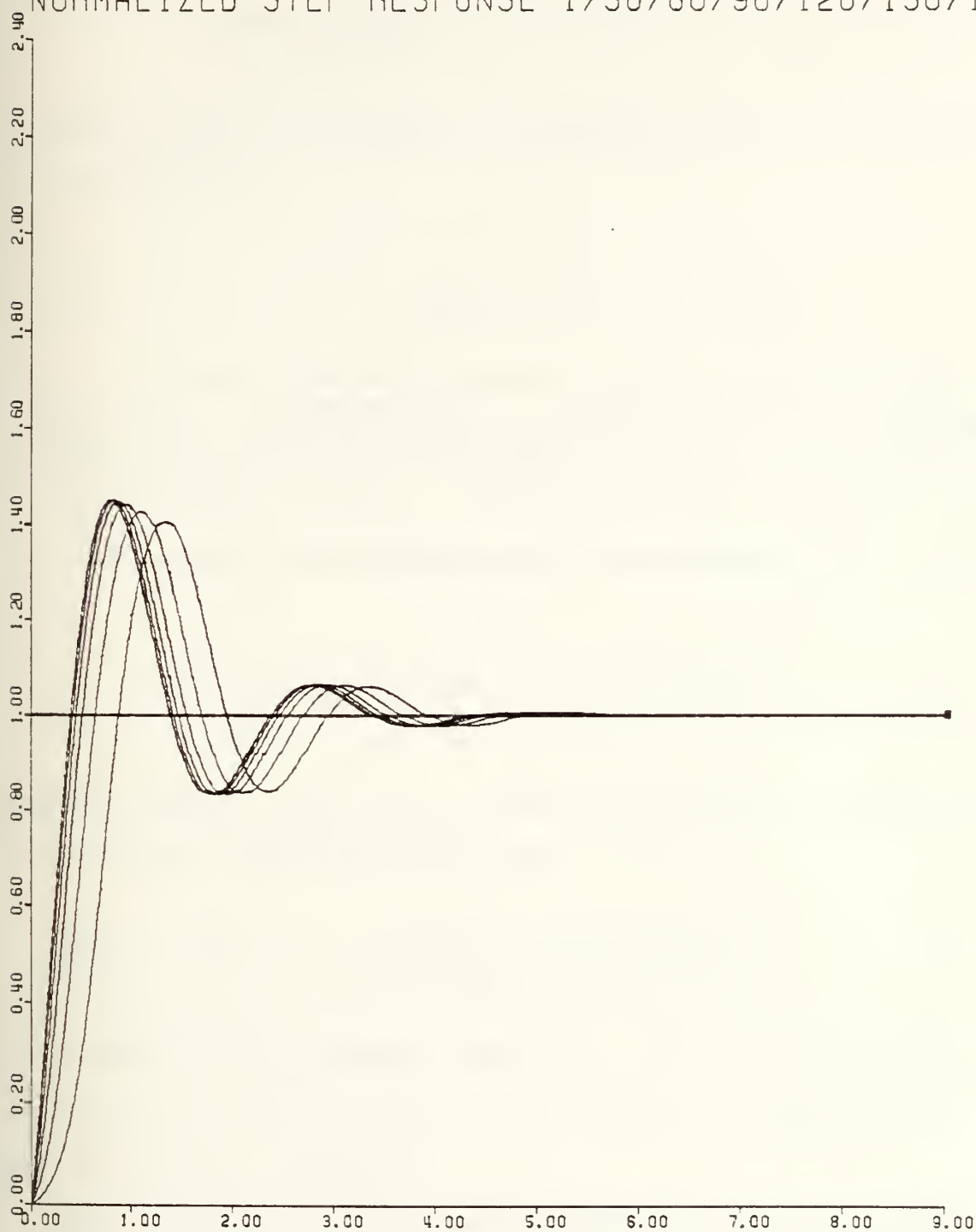
RUN NO. 1  
PLOT NO. 1

Figure 5.4. PLL Step Response, Linear Region





2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION  
 NORMALIZED STEP RESPONSE 1/30/60/90/120/150/170



XSCALE= 1.00  
 YSCALE= 0.20

UNITS/INCH  
 UNITS/INCH

RUN NO. 1  
 PLOT NO. 1

Figure 5.5. PLL Step Response, Non-Linear Region



The forward Euler integrator takes the form,

$$\frac{1}{s} \Longleftrightarrow \frac{Tz^{-1}}{1-z^{-1}}, \quad T = .01 \quad (5.8)$$

The block diagram of Figure 5.3 changes character to appear as in Figure 5.6.

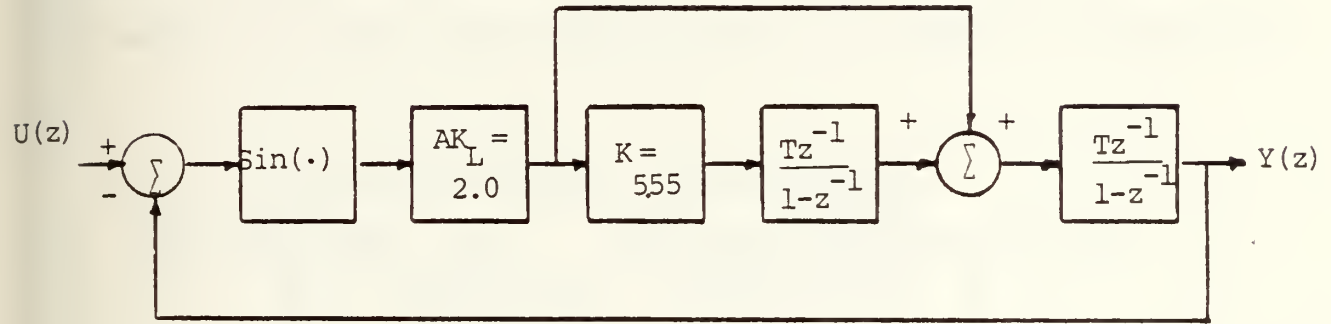


Figure 5.6. Discrete PLL Block Diagram

Applying Mason's gain rule to the block diagram of Figure 5.6 in the linear region we arrive at,

$$\frac{Y(z)}{U(z)} = H(z) = \frac{0.02z^{-1} - 0.018890z^{-2}}{1 - 1.980z^{-1} + .981110z^{-2}} \quad (5.9)$$

the linearized PLL transfer function. The discrete PLL system has zeros at (0.0, .9445) and a complex conjugate pole pair at  $(0.990 \pm j.03178)$ .

The discrete PLL system was implemented using a Fortran program on the IBM 370 in double precision. The source code for the discrete PLL is included as Appendix C. The discrete



PLL was tested using various magnitude step inputs. Results were similar to those from the DSL simulation. That is, the discrete PLL exhibited the same approximate overshoot, rise time, natural frequency and nearly identical time of max overshoot in the linear region. To implement the PLL using forward Euler integration, the block diagram of Figure 5.6 was reconfigured to appear as Figure 5.7. Note that Figure 5.7 does not employ any delay free loops, hence it is easily programmable.

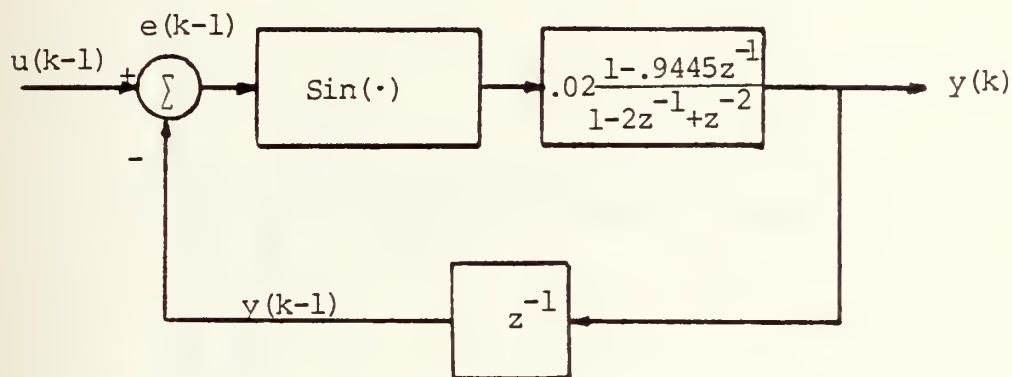


Figure 5.7. Programmable PLL Implementing Forward Euler Integration

Figure 5.8 summarizes the results of the discrete PLL simulation for normalized step inputs in the range  $0 < u(t) \leq 30$ . Similarly, the discrete PLL was tested in the nonlinear region using normalized step inputs in the range  $30 \leq u(t) \leq 170$ . The discrete PLL displayed the same characteristics of decreased output amplitude and delay which was first observed in the DSL simulation. Figure 5.9 summarizes the test results of the discrete PLL in the nonlinear region.



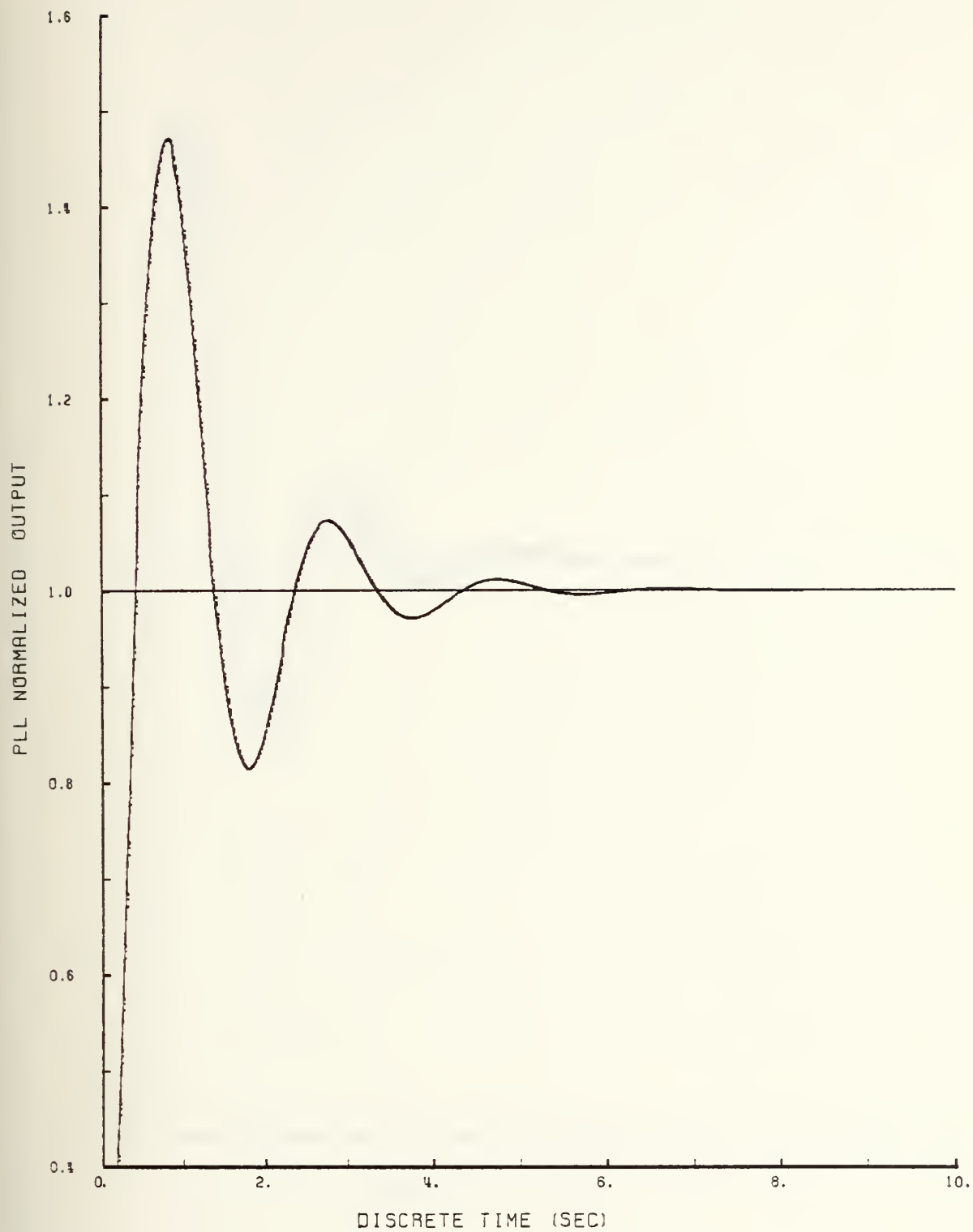


Figure 5.8. Discrete PLL Step Response, Linear Region





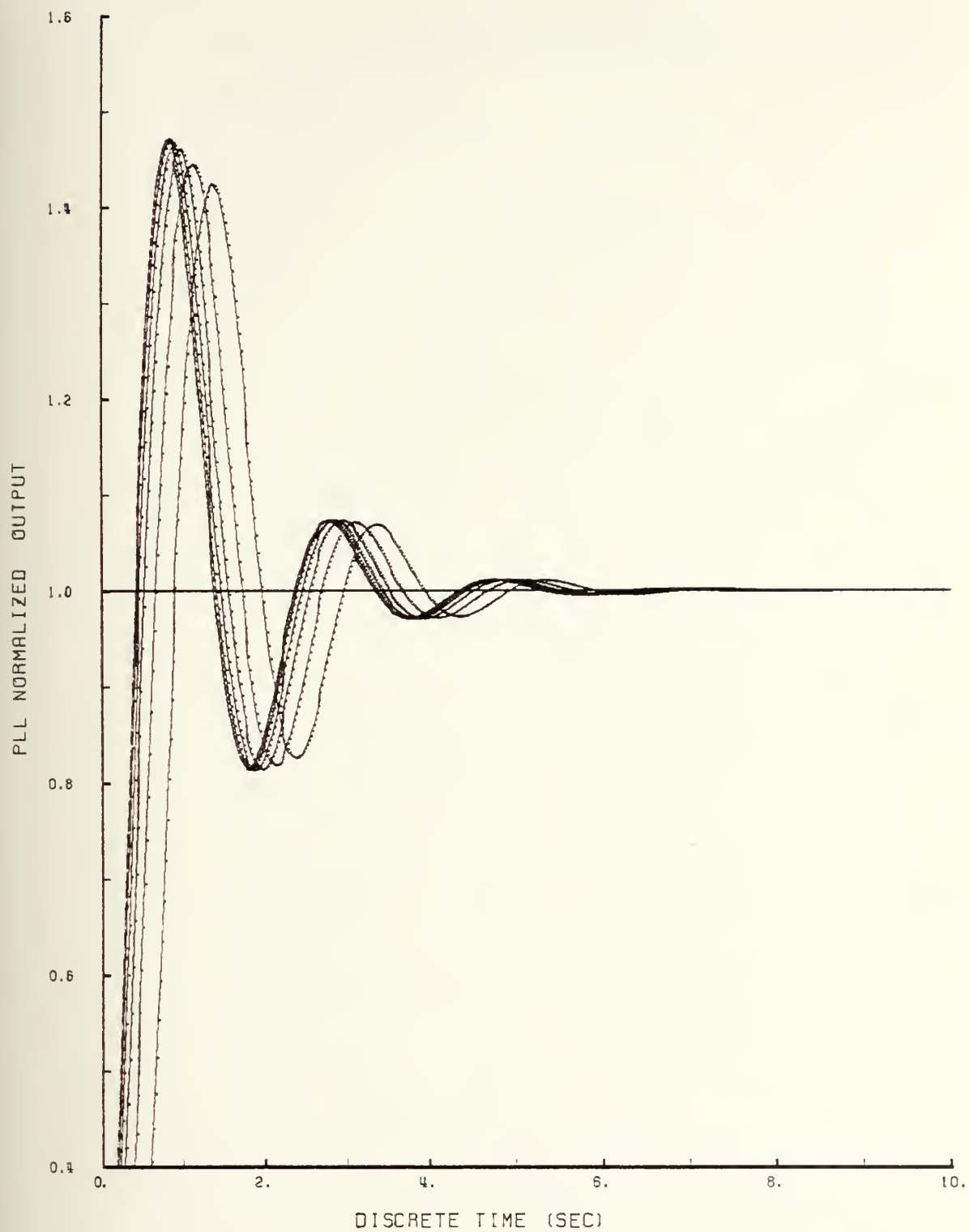


Figure 5.9. Discrete PLL Step Response, Nonlinear Region



It was felt that the PLL was adequately modeled for use as a source of data to be subsequently analyzed by the Adaptive Kalman Identifier.



## VI. MODEL IDENTIFICATION SOFTWARE

Three basic Fortran IV programs for use on the IBM 370 were developed:

- (1) ADAPTSN: Adaptive Kalman Identifier
- (2) LMS: Adaptive Least Mean Square Filter
- (3) LMSR: Adaptive Recursive Least Mean Square Filter

All three programs used double precision arithmetic to minimize the effects of truncation error, limit cycles and roundoff error. In all cases the orders of the autoregressive and moving average processes were read in unformatted form from a disk file along with the input/output data of the unknown system to be analyzed. An attempt was made to minimize memory storage, but not at the expense of program flow and clarity.

### A. ADAPTIVE KALMAN IDENTIFIER

Program ADAPTSN is a versatile Fortran IV software program which implements equations (2.15) to identify the coefficients of an MA, AR, or ARMA process. Additionally, at each iteration the poles and zeros of the evolving transfer function are computed. ADAPTSN can also be used to perform regression analysis on non-linear terms as discussed in Chapter VII. The versatility of the AKI lies in the various options which are available to the user. By properly selecting the flags N, M, and NL, the user can perform regression analysis on data whose combined order  $(N+M+1)$  is less than or equal to 20. Table 6.1 lists the ascribed meanings of the various flags.



Table 6.1

AKI FLAG MEANINGS

- N: the order of the autoregressive process
- M: one greater than the order of the moving average process
- NL = 0: AKI is used to identify the coefficients associated with the general ARMA equation (2.1b) (linear regression)
- NL = 1: AKI is used to identify the coefficients of the linear general ARMA equation and the weighting coefficient associated with  $u^3(k-i)$
- NL = 2: AKI is used as in NL = 0,1 and additionally identifies the weighting coefficient associated with  $y^3(k-i)$
- NL = 3: AKI is used as in NL = 0,1,2 and additionally identifies the weighting coefficient associated with  $u^2(k-i)y(k-i)$
- NL = 4: AKI is used as in NL = 0,1,2,3, and additionally identifies the weighting coefficient associated with  $y^2(i-i)u(k-i)$
- NL = 5: AKI is implemented to analyze time series and identify the ARMA coefficients associated with the Box-Jenkins model [Ref. 26].

The option given by NL = 5 was not extensively tested and as such only limited results are available. Table 6.2 outlines the allowable flag combinations which will produce a valid analysis of the data.

For purposes of this thesis, the non-linear (NL) options were configured to implement the expansion terms associated with the Taylor series expansion of the sine function. This, however, can be changed at the user's discretion to implement any other series expansion by inserting the appropriate Fortran statements at the proper location in the AKI program.





Table 6.2

## VALID FLAG COMBINATIONS

OPTION (NL)	MAX COMBINED ORDER
0	$N + M \leq 20$
1	$N + 2M \leq 20; \quad N, M \neq 0$
2	$2N + 2M \leq 20; \quad N, M \neq 0$
3	$3N + 2M \leq 20; \quad N, M \neq 0$
4	$4N + 2M \leq 20; \quad N, M \neq 0$
5	$N + M \leq 20$

The overall structure of the AKI program uses subroutine calls to compute the various quantities necessary for the eventual computation of the system ARMA coefficients. These subroutines are: GAIN, RECKON, PRINT, NEXT. A brief description of the function each performs is given at the beginning of each subroutine. Figure 6.1 describes the general program flow of the AKI. The source code for program ADAPTSN is included as Appendix D.

#### B. ADAPTIVE LEAST MEAN SQUARE FILTER

Program LMS uses the equations developed in Chapter III to compute the coefficients (or weights) associated with a moving average process. The LMS filter program is capable of computing up to 12 MA coefficients; or equivalently, is capable of identifying an eleventh order moving average process. The general program flow of the LMS filter is presented in Figure 6.2 and the annotated source code is included as Appendix E.



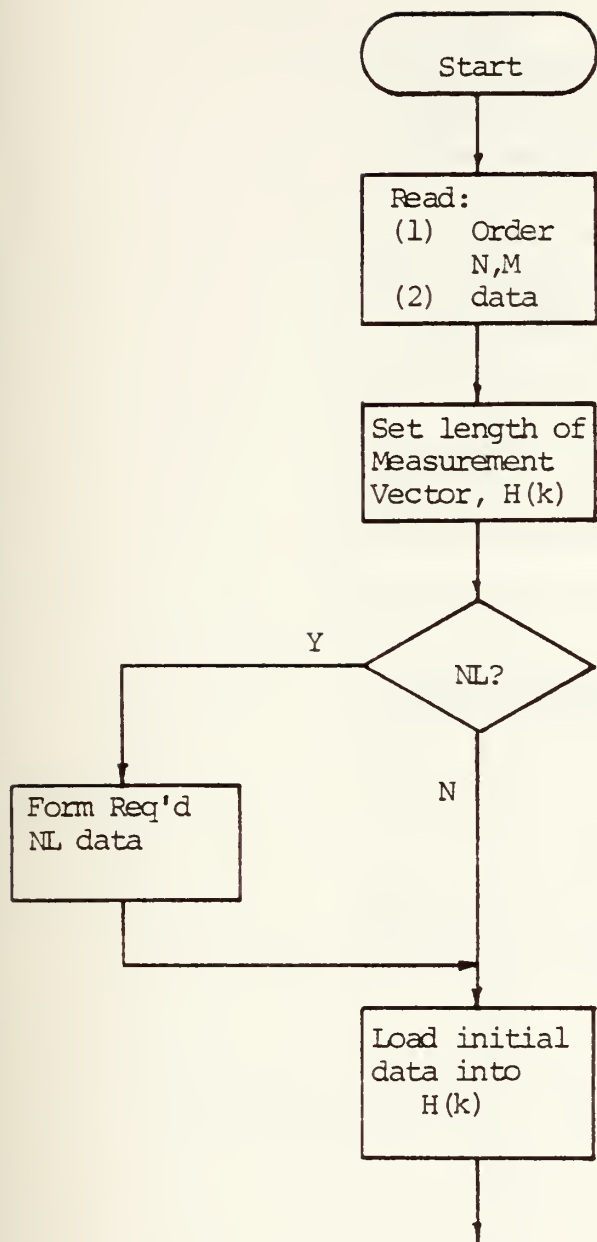


Figure 6.1. AKI General Program Flow



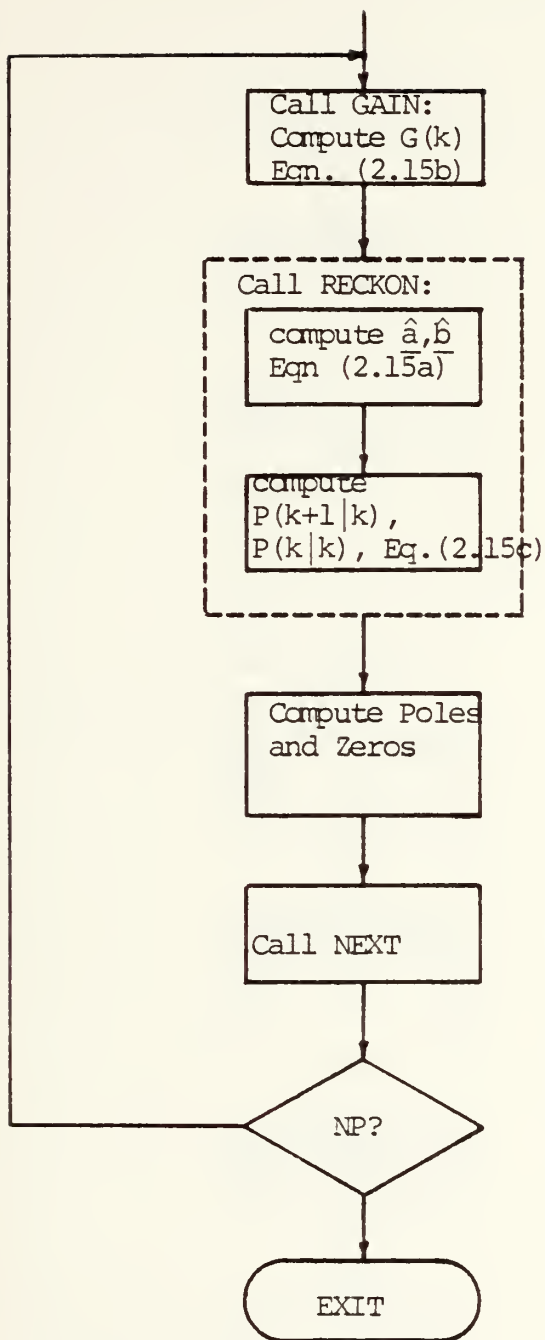


Figure 6.1. (CONTINUED)



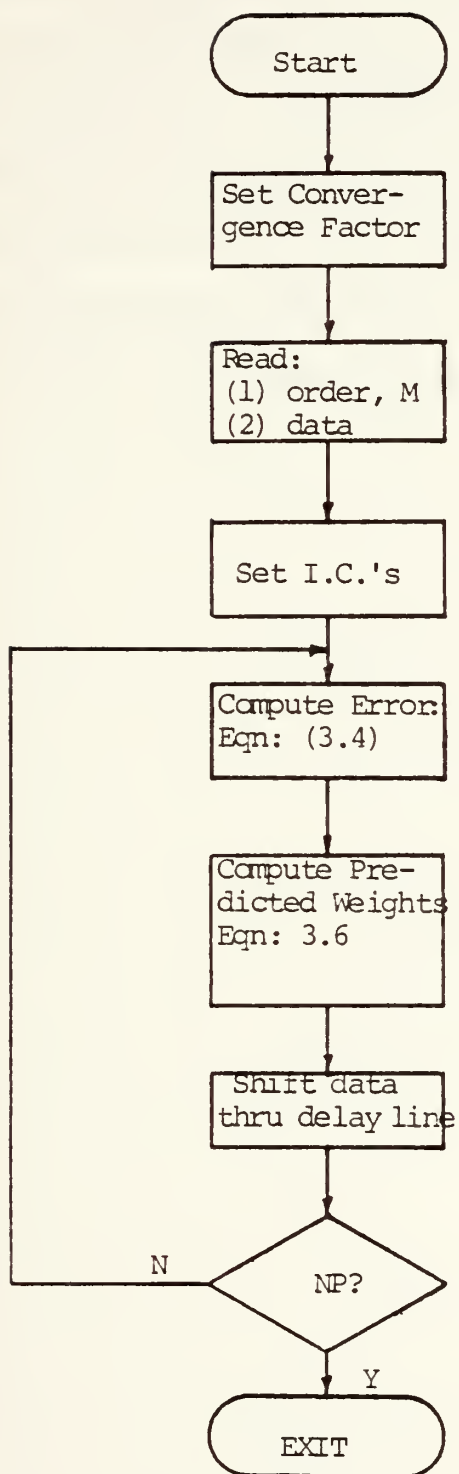


Figure 6.2. LMS Filter Program Flow





### C. ADAPTIVE RECURSIVE LEAST MEAN SQUARE FILTER

Program LMSR realizes the algorithm proposed by Feintuch and recapped in Chapter IV. It was designed to handle a combined AR and MA order of 11. That is, the program can be used to estimate  $m + 1 = M$  coefficients associated with the MA process and  $n = N$  coefficients associated with the AR process such that  $N + M \leq 12$ . The general program flow for the Adaptive Recursive LMS filter is shown in Figure 6.3 and the source code is included as Appendix F.



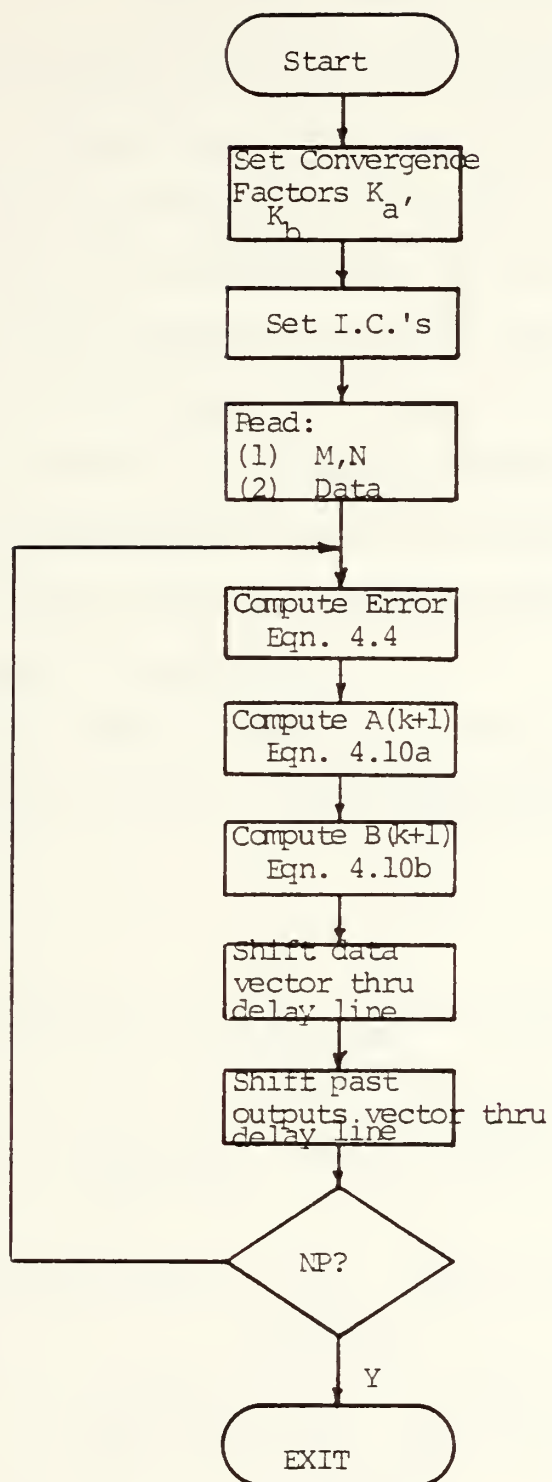


Figure 6.3. LMSR Filter Program Flow



## VII. NON-LINEAR IDENTIFICATION

The Adaptive Kalman Identifier can be modified to estimate the weighting coefficients associated with higher order terms produced by some non-linearity within the unknown system. In general, much has to be known about the non-linearity such that the functional description chosen for it is a close approximation to the effects it causes. In this thesis one possible application of the Adaptive Kalman Identifier toward identifying a system with a known non-linearity is explored. No attempt has been made to present an extensive treatment of non-linear analysis techniques.

The approach taken has been previously explored by Parker [Ref. 8]. A special case of the generalized non-linear ARMA model [Ref. 8],

$$\begin{aligned}
 y(k) = & \sum_{i_1=0}^{\infty} a(i_1)u(k-i_1) + \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a(i_1, i_2)u(k-i_1)u(k-i_2) + \dots \\
 & + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} a(i_1, \dots, i_m)u(k-i_1) \dots u(k-i_m) \\
 & + \sum_{j_1=1}^{\infty} b(j_1)y(k-j_1) + \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{\infty} b(j_1, j_2)y(k-j_1)y(k-j_2) + \dots \\
 & + \sum_{j_1=1}^{\infty} \dots \sum_{j_n=1}^{\infty} b(j_1, j_2, \dots, j_n)y(k-j_1)y(k-j_2) \dots y(k-j_n) \\
 & + \sum_{i_j=0}^{\infty} \sum_{j_1=1}^{\infty} c(i_1, j_1)u(k-i_1)y(k-j_1) + \dots + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} \sum_{j_1=1}^{\infty} \\
 & \dots \sum_{j_n=1}^{\infty} c(i_1, \dots, i_m, j_1 \dots j_n)u(k-i_1) \dots u(k-i_m)y(k-j_1) \dots y(k-j_n)
 \end{aligned}
 \tag{7.1}$$



is used to model the input/output relationships, non-linear transfer function, of the phase locked loop. The non-linear element, the sine function, of the PLL can be replaced by a Taylor power series expansion,

$$\sin(x) = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \dots \quad (7.2)$$

Other expansions can be used, e.g., Legendre polynomials, Volterra series, ..., etc.; however, only the Taylor expansion was investigated in this thesis. Practical implementation of (7.2) suggests truncation at the third order term. The sine function is therefore approximated as

$$\sin(x) = x - \frac{1}{3!} x^3. \quad (7.3)$$

Substituting (7.3) into the block diagram of Figure (5.6) we have Figure 7.1.

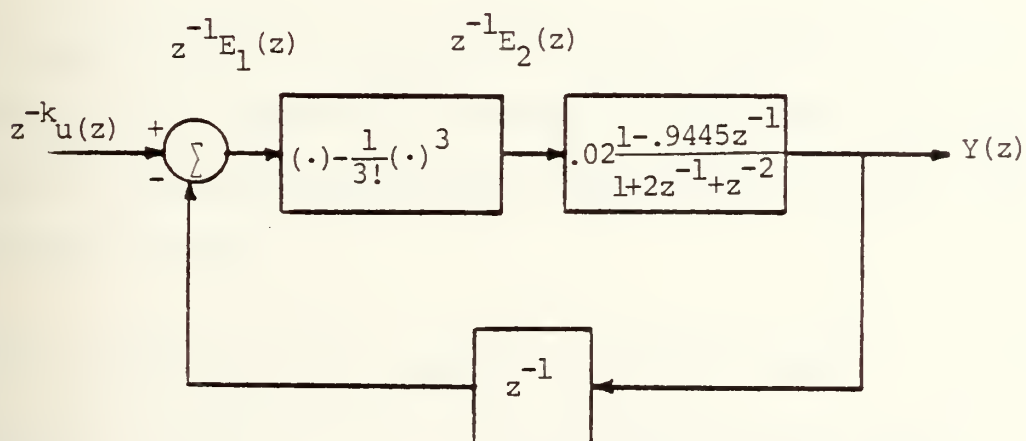


Figure 7.1. PLL, Third Order Taylor Series Approximation





The discrete time domain equations at the different nodes are:

$$e_1(k-1) = u(k-1) - y(k-1) \quad (7.4a)$$

$$e_2(k-1) = e_1(k-1) - \frac{1}{3!} e_1^3(k-1) \quad (7.4b)$$

$$\begin{aligned} y(k) &= .02e_2(k-1) - .01889e_2(k-2) + 2.0y(k-1) \\ &\quad - y(k-2) \end{aligned} \quad (7.4c)$$

Manipulating equations (7.4a)-(7.4c) gives

$$\begin{aligned} y(k) &= .02u(k-1) - .01889u(k-2) + 1.98y(k-1) - .98111y(k-2) \\ &\quad - .003333u^3(k-1) + .003148333u^3(k-2) \\ &\quad + .003333y^3(k-1) - .003148333y^3(k-2) \\ &\quad + .01u^2(k-1)y(k-1) - .09445u^2(k-2)y(k-2) \\ &\quad - .01u(k-1)y^2(k-1) + .09445u(k-2)y^2(k-2). \end{aligned} \quad (7.5)$$

Equation (7.5) indicates which non-linear terms of equation (7.1) should be retained. Therefore, the third order non-linear approximation model should contain the following terms:

$$u(k-1), u(k-2), y(k-1), y(k-2) \quad \text{linear terms} \quad (7.6a)$$

$$u^3(k-1), u^3(k-2) \quad \text{input cubic terms} \quad (7.6b)$$

$$y^3(k-1), y^3(k-2) \quad \text{output cubic terms} \quad (7.6c)$$



$$\left. \begin{array}{l} u^2(k-1)y(k-1), u^2(k-2)y(k-2) \\ y^2(k-1)u(k-1), y^2(k-2)u(k-2) \end{array} \right\} \begin{array}{l} \text{third order cross} \\ \text{product terms} \end{array} \quad (7.6d)$$

The AKI algorithm is primarily modified to include the hybrid signals (7.6) in the structure of  $H(k)$ , the measurement vector. Since now the measurement vector,  $H(k)$  takes the form,

$$\begin{aligned} H(k) = & [u(k) \dots u(k-m), -y(k-1) \dots -y(k-n), u^3(k) \dots \\ & \dots u^3(k-m), -y^3(k-1) \dots -y^3(k-n), -(u^2(k-1)y(k-1)) \dots \\ & \dots -(u^2(k-n)y(k-n)), (u(k-1)y^2(k-1)) \dots (u(k-n)y^2(k-n))] \end{aligned} \quad (7.7)$$

the AKI algorithm calculates the coefficients associated with the special case of the generalized non-linear ARMA model, equation (7.1).

Romeo using a similar approach [Ref. 7] computes a least squares curve fit for the third order truncation of the Taylor series for the sine function over the interval  $(0, \pi/2)$ . Beginning with the approximation,

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.8)$$

Romeo finds the values for  $\alpha$  and  $\beta$  to be .865 and -.095 respectively. Performing the same operations as those used in obtaining equation (7.5) we arrive at,



$$\begin{aligned}
y(k) = & .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\
& + (.01889\alpha - 1)y(k-2) - .02\beta u^3(k-1) + .01889\beta u^3(k-2) \\
& - .01889\beta y^3(k-2) + .06\beta u^2(k-1)y(k-1) + .02\beta y^3(k-1) \\
& - .056670\beta u^2(k-2)y(k-2) - .06\beta y^2(k-1)u(k-1) \\
& + .056670\beta y^2(k-2)u(k-2)
\end{aligned} \tag{7.9}$$

Table 7.1 tabulates the resulting weighting coefficients using the three methods just described: (1) analytically calculated values using the truncated Taylor series, (2) least squares estimate of the third order sine approximation, as computed by Romeo [Ref. 7] and (3) the coefficient estimates as computed using the AKI algorithm.

The tabulation clearly shows that the AKI outperforms the first two methods overall. That is, the linear terms were identified without question; however, the AKI failed to identify the coefficients associated with the  $y^3(i-1)$ ,  $y^3(k-2)$ ,  $u^2(k-2)y(k-2)$  and  $y^2(k-2)u(k-2)$  terms. The reason for the failure is not known and was not investigated.



Table 7.1

NON-LINEAR WEIGHTING COEFFICIENTS CALCULATED  
USING THREE METHODS

(1)		(2)	(3)
TAYLOR SERIES		LEAST SQUARES EST.	AKI USING ACTUAL
$\sin(x) = x - \frac{1}{3!}x^3$		$\sin(x) = \alpha x + \beta x^3$ $\alpha = .865, \beta = -.095$	$\sin(x)$ data base
y(k-1)	1.98	1.982700	1.980
y(k-2)	-.98111	-.98366015	-.9815
u(k)	0.0	0.0	.00003691
u(k-1)	0.2	.01730000	.02003
u(k-2)	-.01889	-.01633985	-.01892
u <sup>3</sup> (k)	0.0	0.0	.00001471
u <sup>3</sup> (k-1)	-.003333	.00190	-.003278
u <sup>3</sup> (k-2)	.003148333	-.00179455	.003032
y <sup>3</sup> (k-1)	.003333	-.00190	-.05695
y <sup>3</sup> (k-2)	-.003148333	.00179455	.05799
u(k-1)y(k-1)	.01	-.005700	.01032
u <sup>2</sup> (k-2)y(k-2)	-.09445	.00538365	-.009610
y <sup>2</sup> (k-1)y(k-1)	-.01	.005700	-.01640
y <sup>2</sup> (k-2)u(k-2)	.09445	-.00538365	.01913





## VIII. FINDINGS AND CONCLUSION

The performance of the Adaptive Kalman Identifier was compared to the LMS Adaptive Filter and the Adaptive Recursive LMS Filter using data derived from the models discussed in Chapter V. Results for the case where the order of the model (that is,  $m$  and  $n$  of equation (2.1b)) are assumed known are presented first followed by the analysis of the PLL data. The findings for the overmodeled case are presented next. The graphs show typical runs and do not represent ensemble averages.

### A. ORDER OF THE UNKNOWN SYSTEM IS KNOWN

#### 1. AKI vs. LMS Adaptive Filter

Using the MA form of the Adaptive Kalman Identifier, its performance can be compared against that of the LMS adaptive filter. Synthetic data derived from a plant whose transfer function is,

$$H(z) = 1.0 + 2.0z^{-1} + 3.0z^{-2} \quad (8.1)$$

was used. However, a fair comparison necessitated that the LMS Adaptive filter be "tuned" by adjustment of the convergence factor,  $k_s$ . Several values for  $k_s$  in the range  $[-.600, -.200]$  were used. The objective of tuning the LMS filter was to achieve a fast convergence time with little or no steady state error while not compromising filter stability. The three filter weights were normalized and plotted for five



convergence factor values. It can be seen from Figures 8.1-8.3 that convergence is essentially reached by step thirty-five, ( $k \approx 35$ ). As the convergence factor is further increased it can be noted in Figure 8.4 that the filter weights become more noisy, and that when  $k_s = -.600$ , filter stability is being compromised (Figure 8.5a).

Using the same data, the AKI converges to the MA coefficients in less than five iterations. Referring to Figure 8.5b, it was also noted that as the measurement noise  $v(k)$  was increased, the LMS algorithm yielded more noisy estimates whereas the AKI tended to compensate for measurement noise. This is intuitively reasonable since the AKI incorporates into its algorithm the effects of measurement error due to noisy sensors. Comparing Figure 8.6 with Figure 8.7 the latter figure clearly shows the faster convergence of the AKI to the plant coefficient.

It was shown in Chapter III that the AKI gains would approach a steady state value and indeed they did, as Figure 8.8 shows. However, the instantaneous Kalman gains display a more erratic pattern as shown in Figure 8.9. The information of Figure 8.8 was gleaned from Figure 8.9 by computing the average of the individual gains at each iteration using the algorithm,

$$E\{\underline{G}(k+1)\} = E\{\underline{G}(k)\} + \frac{1}{k+1}[\underline{G}(k) - E\{\underline{G}(k)\}] \quad (8.2)$$

where,

$E\{\underline{G}(k+1)\}$  is the one step prediction of the average value for the gains,



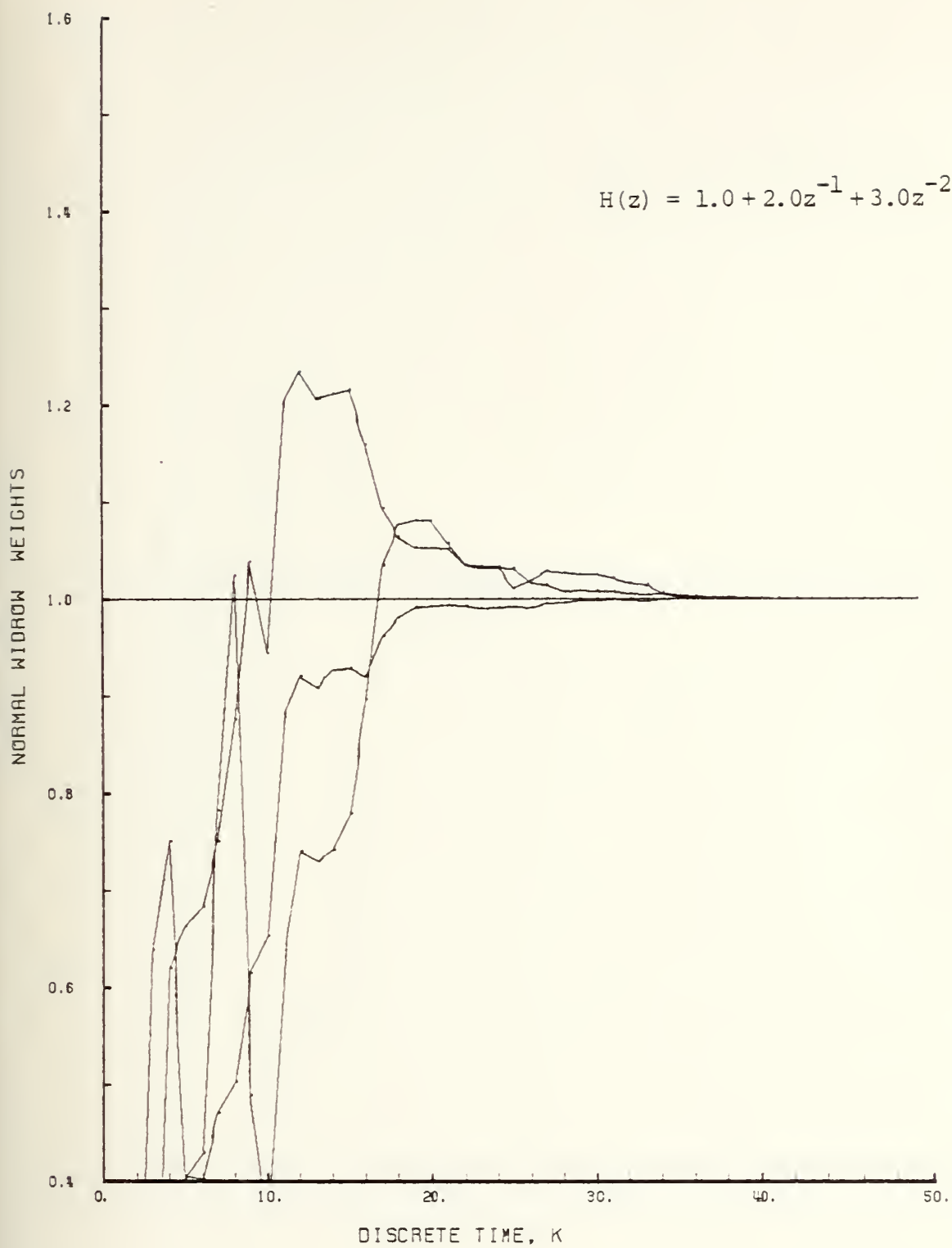


Figure 8.1. LMS Adaptive Filter Weights,  $k_s = -.200$



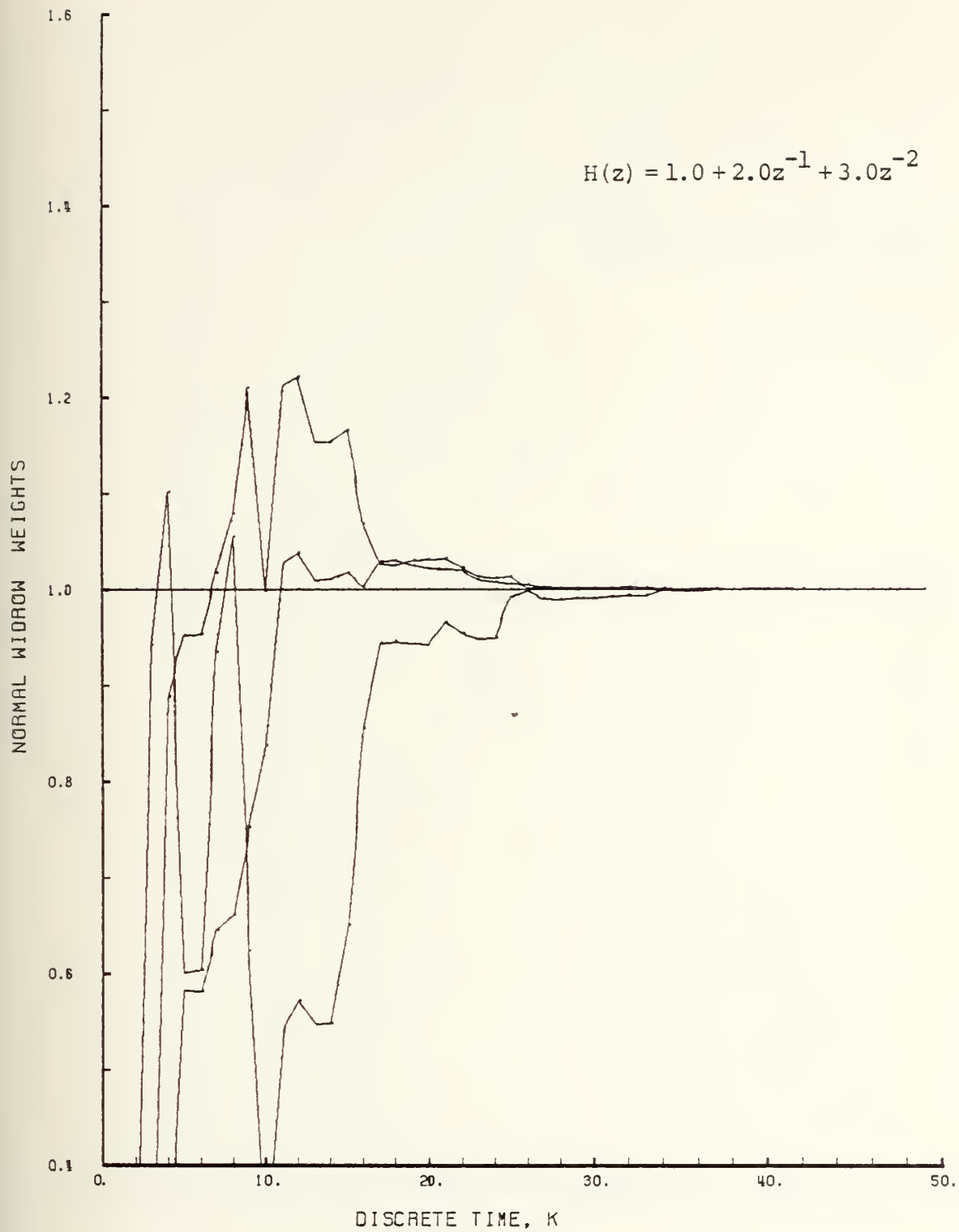


Figure 8.2. LMS Adaptive Filter Weights,  $k_s = -.300$





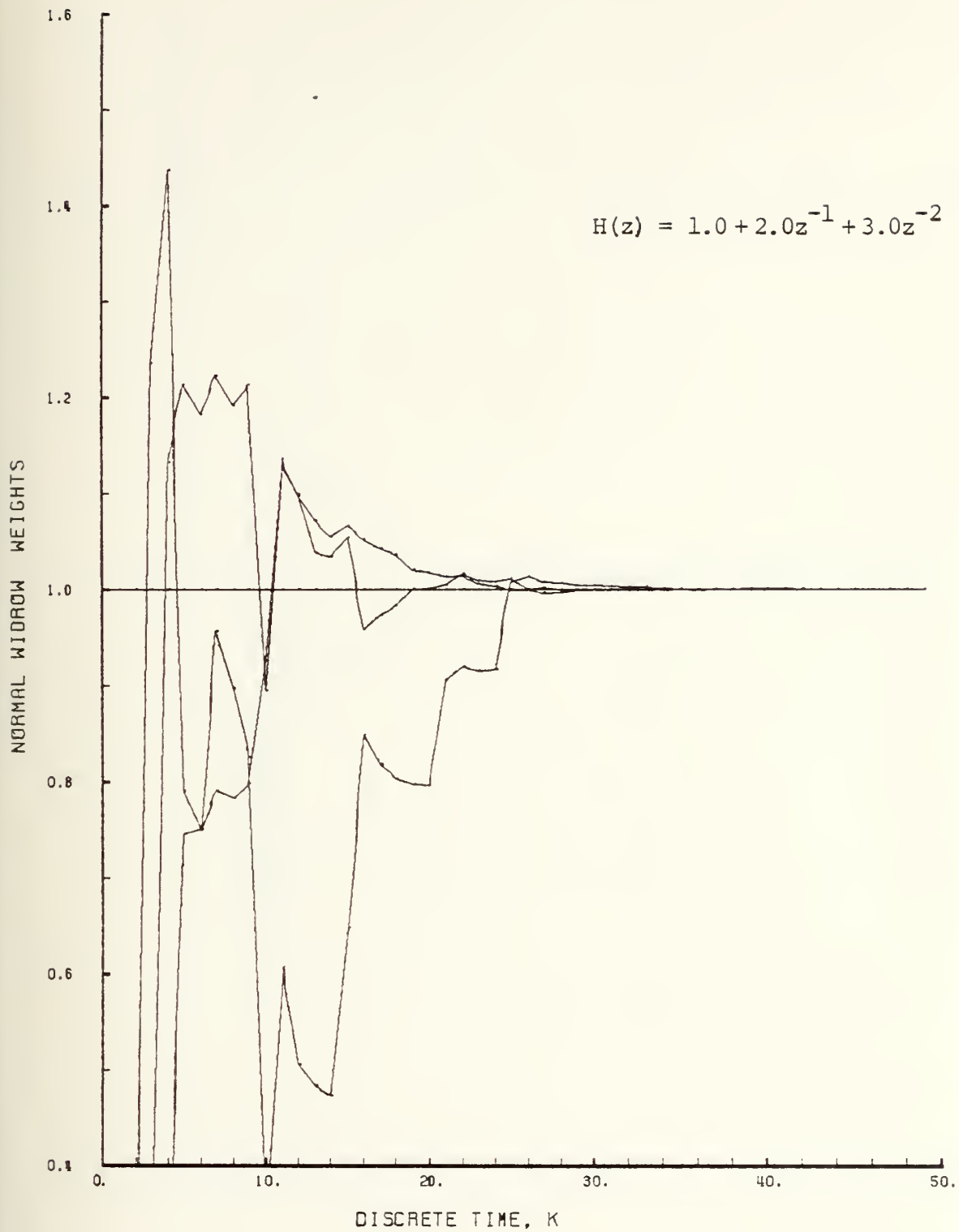


Figure 8.3. LMS Adaptive Filter Weights,  $k_s = -.400$



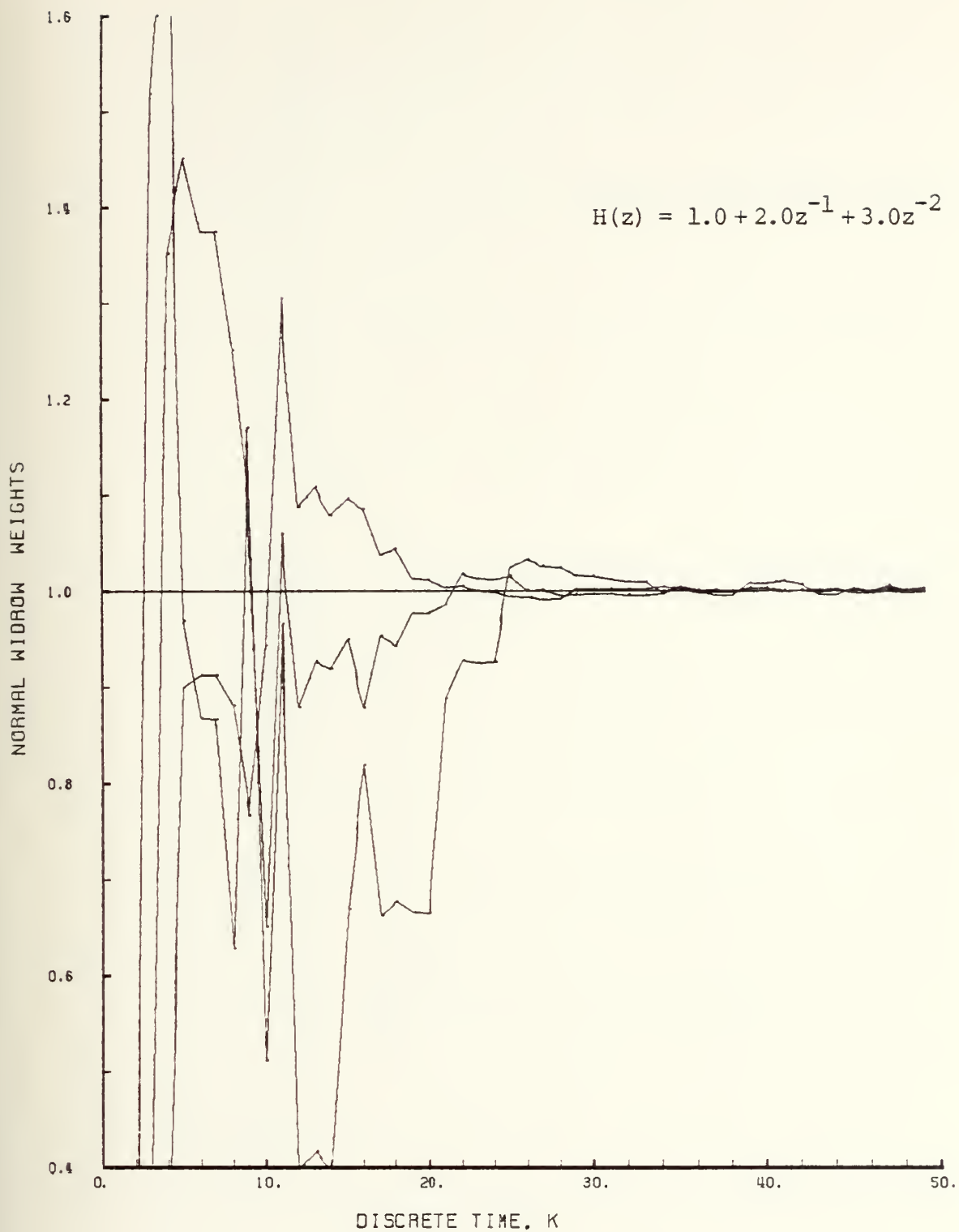


Figure 8.4. LMS Adaptive Filter Weights,  $k_s = -.500$



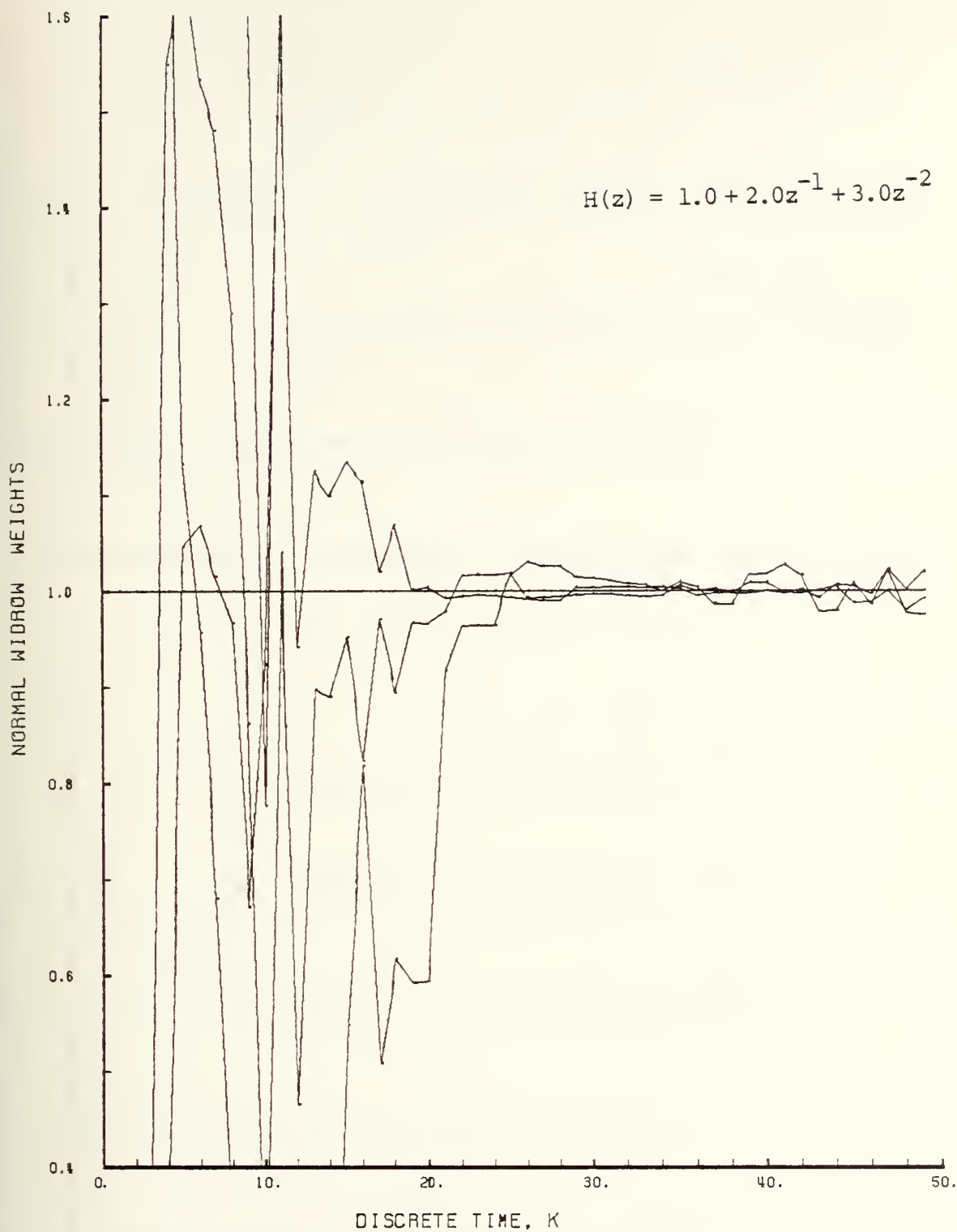


Figure 8.5a. LMS Adaptive Filter Weights,  $k_s = -.600$



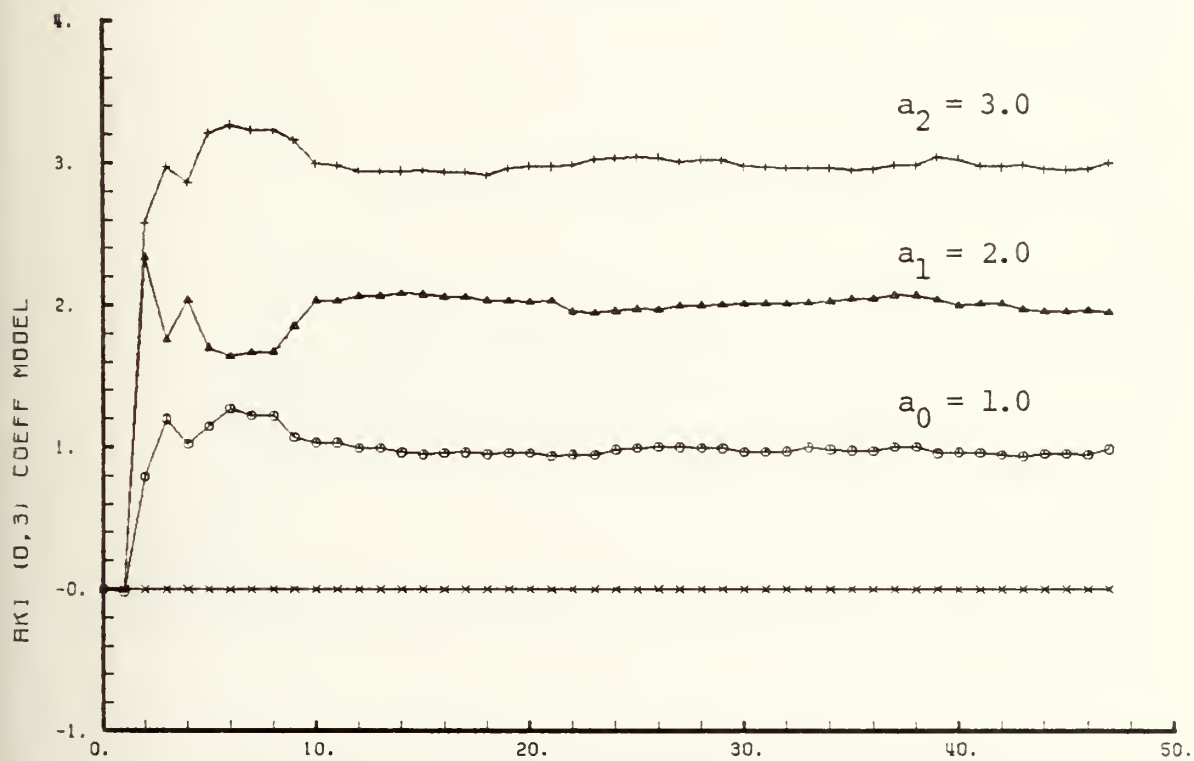
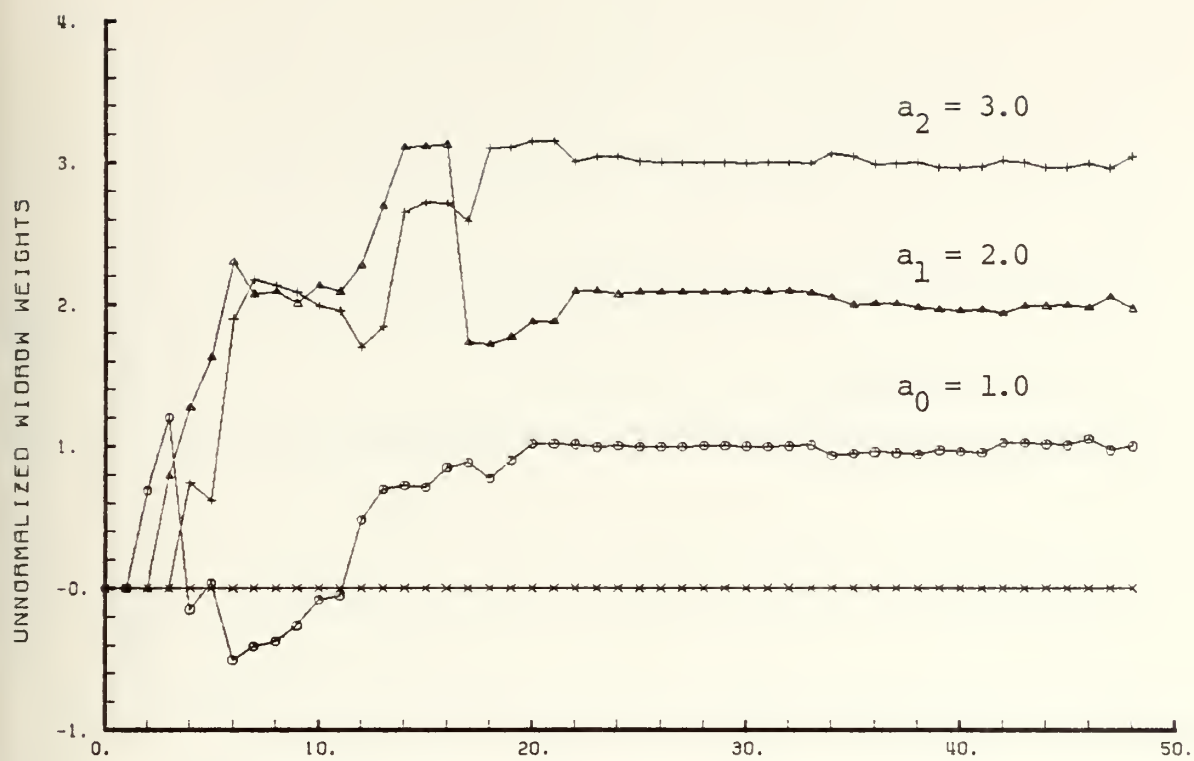


Figure 8.5b. LMS Adaptive Filter Weights,  $k_s = -.200$ , with increased measurement noise





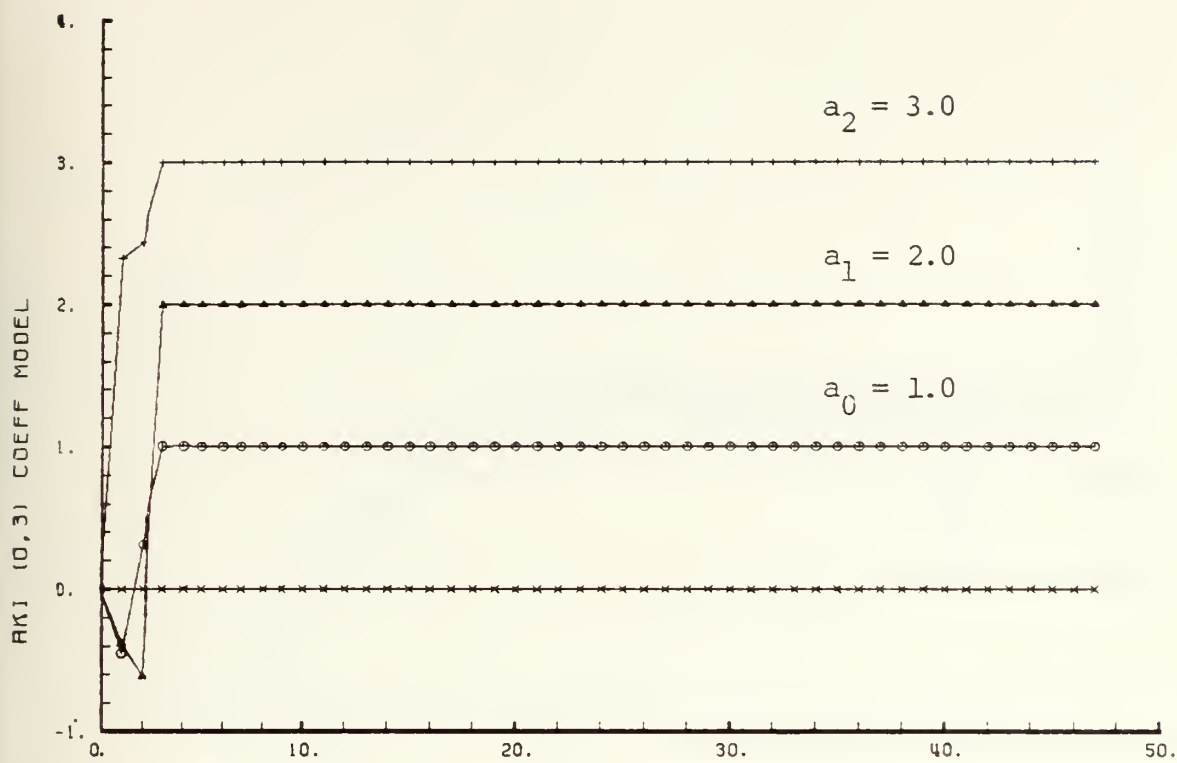


Figure 8.6. AKI Moving Average Plant Coefficients

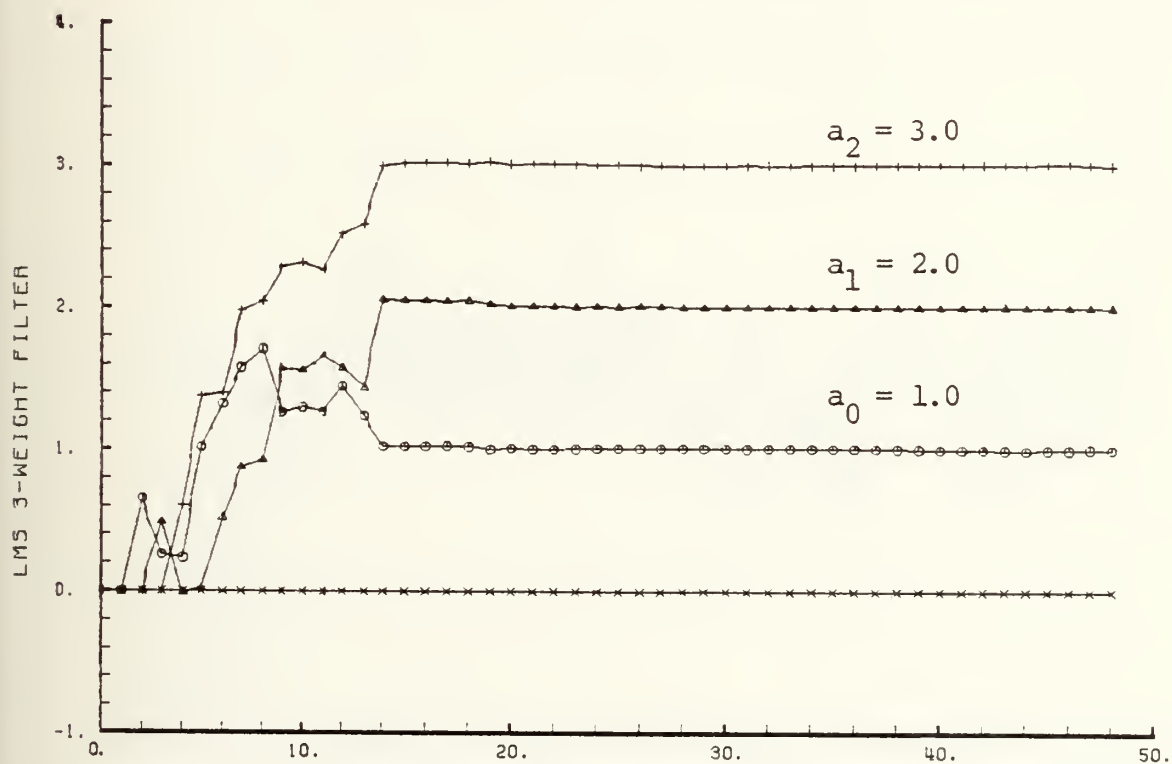


Figure 8.7. LMS Adaptive Filter Weights



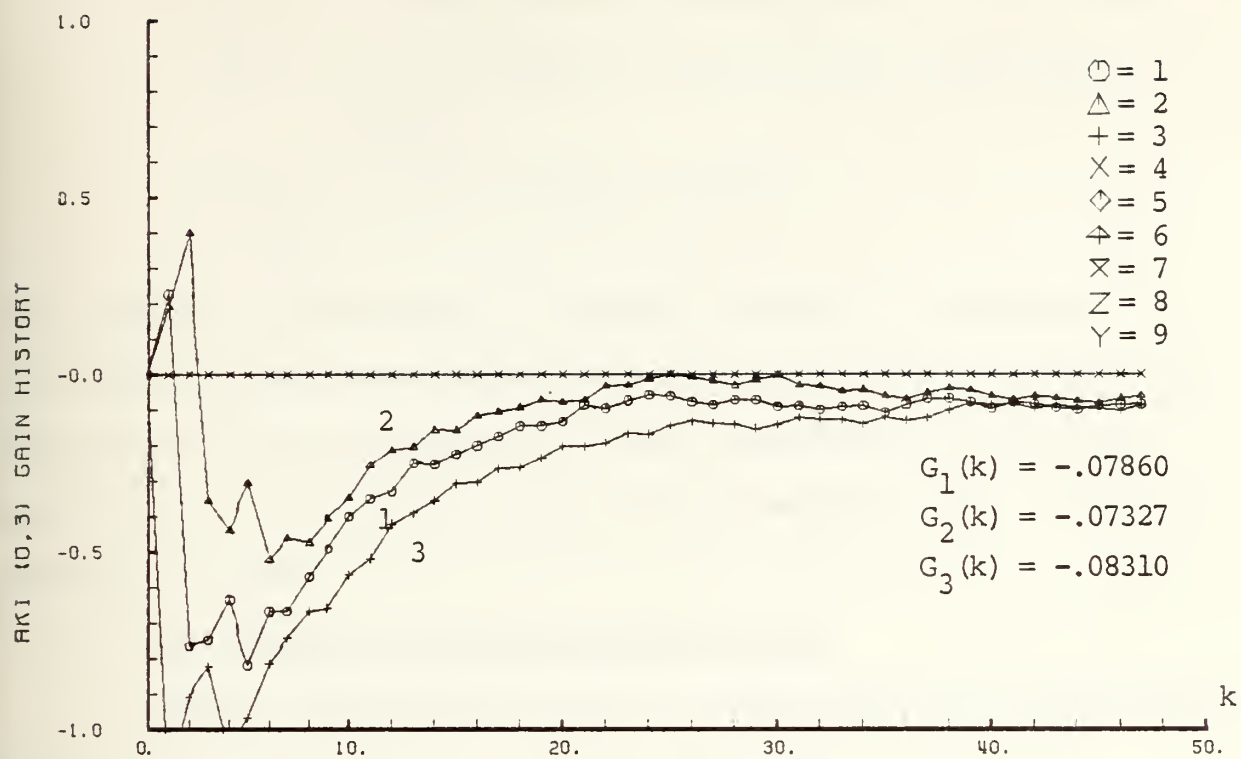


Figure 8.8. AKI Averaged Gains for Moving Average Model

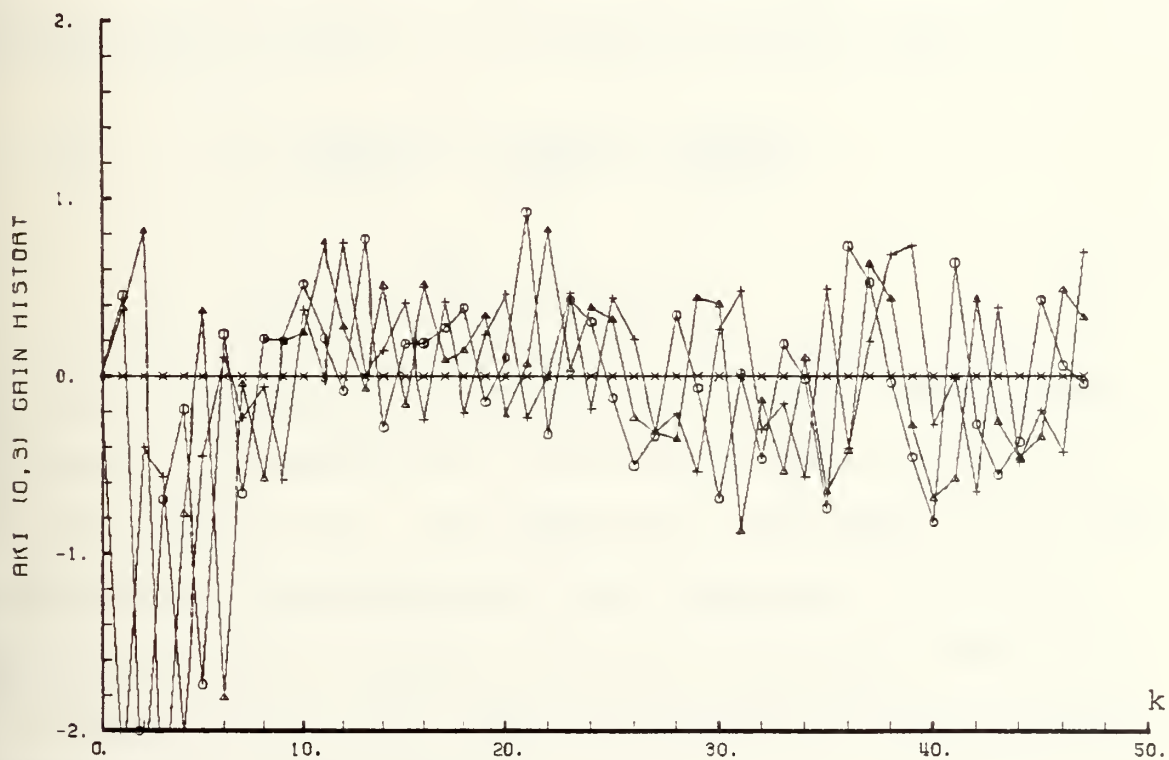


Figure 8.9. AKI Moving Average Instantaneous Gains



$E\{\underline{G}(k)\}$  is the present average value for the gain

$\underline{G}(k)$  is the present value of the instantaneous gains

$\frac{1}{k+1}$  is the averaging gain.

This method of obtaining a "running" average is generally well known, (see for example Ref. 39). The example presented was not the only one used but a good representative of the operation of the AKI when the data was derived from a moving average process.

## 2. AKI Applied to Autoregressive Data

Before applying the AKI to identifying the coefficients of a complex ARMA plant, it was first tested using data derived from a plant whose transfer function specified an autoregressive process. The transfer function used was,

$$\begin{aligned} H(z) &= \frac{1}{1.0 + \frac{26.0}{24.0}z^{-1} + \frac{9.0}{24.0}z^{-2} + \frac{1.0}{24.0}z^{-3}} \\ &= \frac{a_0}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3}} \end{aligned} \quad (8.3)$$

The AKI had no problem converging on the plant coefficients (Figure 8.10) producing the following estimates at the 42<sup>nd</sup> iteration (Table 8.1). It essentially converged on the plant coefficients in approximately five iterations. The gains computed by the AKI for this case also displayed convergence to a steady state value. The gain history for this example is depicted in Figure 8.11.



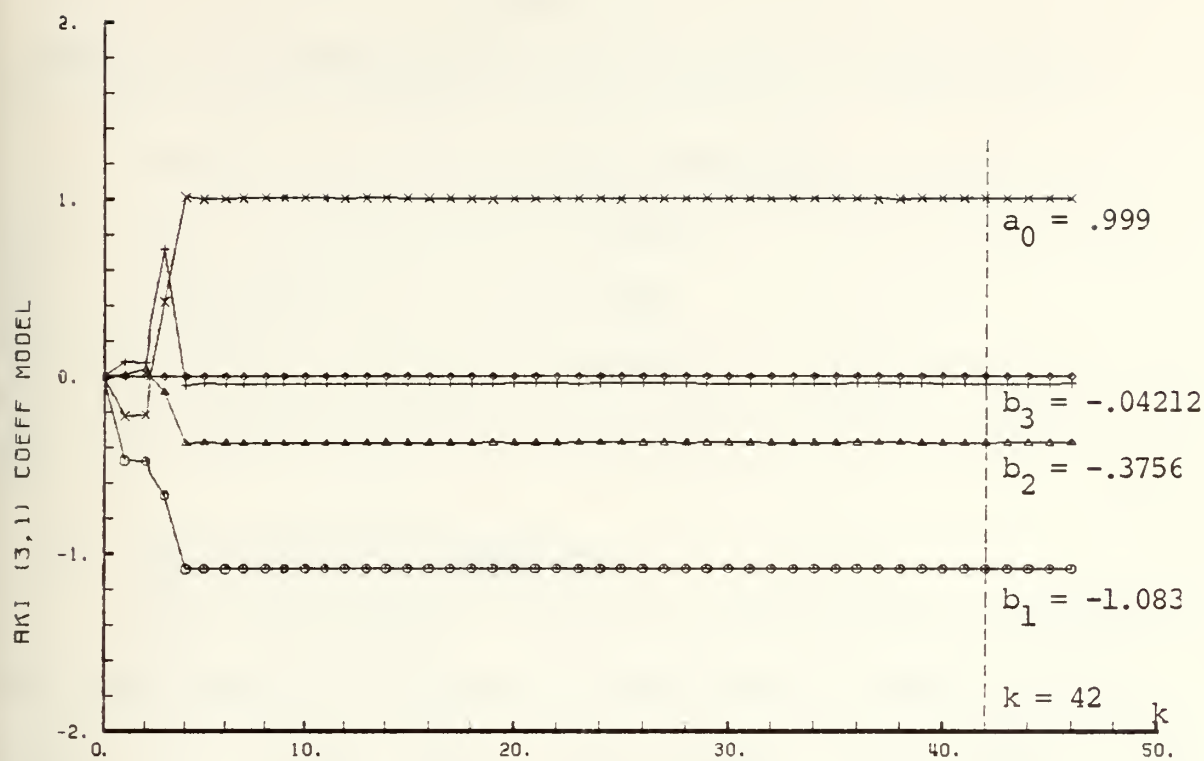


Figure 8.10. AKI Autoregressive Plant Coefficients

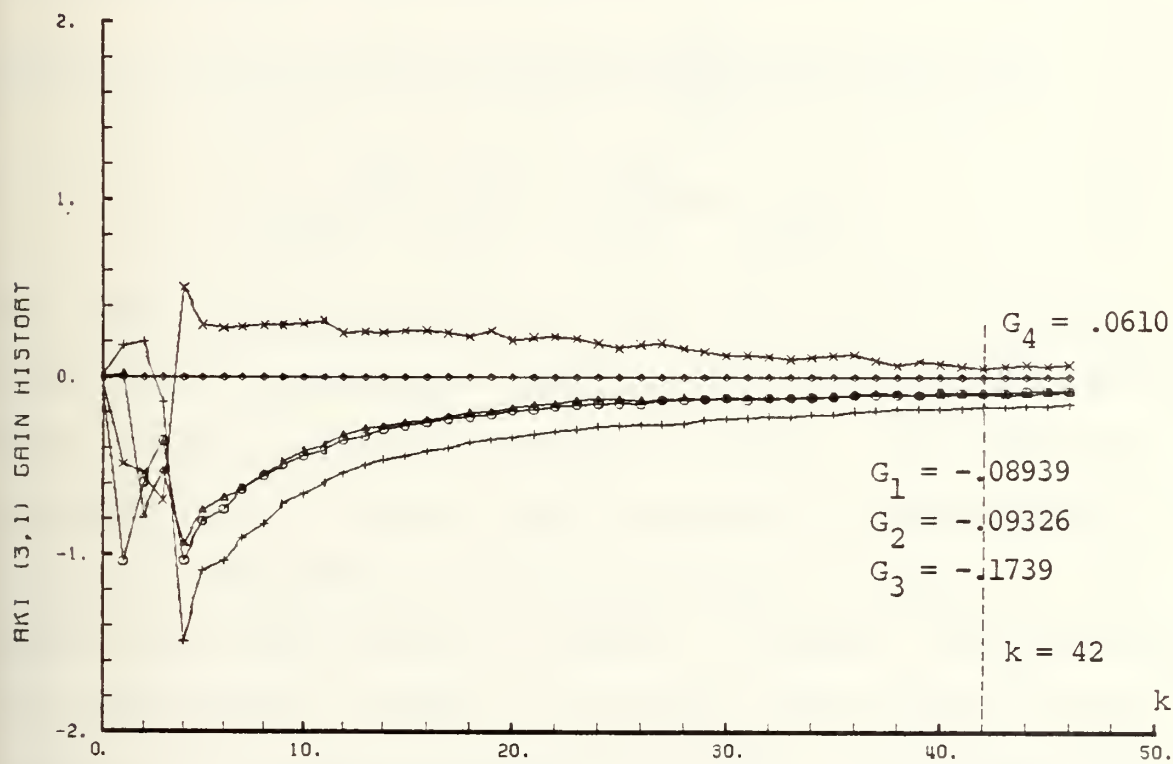


Figure 8.11. AKI Autoregressive Averaged Gains





Table 8.1

## ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION 8.3

	ACTUAL	AKI ESTIMATES
$a_0$	1.0	0.9995
$b_1$	-1.08333 ...	-1.083
$b_2$	-0.3750	-0.3756
$b_3$	-0.04166 ...	-0.04212

3. AKI Applied to ARMA Data

The next logical step was to use the AKI to identify the coefficients of a general ARMA process. One of Perry's models [Ref. 6] was used for this purpose. Specifically, the transfer function of the plant was,

$$\begin{aligned}
 H(z) &= \frac{1.0 + 1.4z^{-1} + .98z^{-2}}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \\
 &= \frac{a_0 + a_1z^{-1} + a_2z^{-2}}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3} - b_4z^{-4}} \quad (8.4)
 \end{aligned}$$

The ARMA plant was subjected to the same conditions which Perry [Ref. 6] describes. That is, unit variance, zero mean, white gaussian noise was used as the input. The reader is reminded that the output signal processed by the AKI was corrupted by measurement noise as described in Chapter V. The output data used in Perry's examples, however, reflects noiseless measurements. Table 8.2 tabulates the results for the coefficient estimates computed by the AKI. Even though the results presented in Table 8.2 represent the coefficient



Table 8.2

ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION  
8.4 AT THE 371ST ITERATION

	ACTUAL	AKI ESTIMATES
$a_0$	1.00000	0.98760
$a_1$	1.40000	1.40100
$a_2$	0.98000	0.99280
$a_3$	0.00000	0.01262
$a_4$	0.00000	-0.00922
$b_1$	1.14000	1.14000
$b_2$	-1.45490	-1.45900
$b_3$	0.88490	.88740
$b_4$	-0.40745	-.40980

estimates at the 371<sup>st</sup> iteration, it can be seen from Figure 8.12 that the AKI has essentially converged by the 28<sup>th</sup> iteration. We note also the characteristic convergence of the averaged gains to steady state values in Figure 8.13. As a means of comparison with Perry's results, the poles and zeros of the AKI estimates at the 28<sup>th</sup>, 90<sup>th</sup> and 371<sup>st</sup> iteration are plotted in Figure 8.14a and Figure 8.14b. Perry's results Figure 3.7 [Ref. 6:pp. 107,108] for his lattice modeling of the plant represented by equation (8.4) are reproduced for convenience.

As was noted for the previous cases, the instantaneous gains appeared erratic, Figure 8.15, whereas the averaged gains converged to some steady state value.



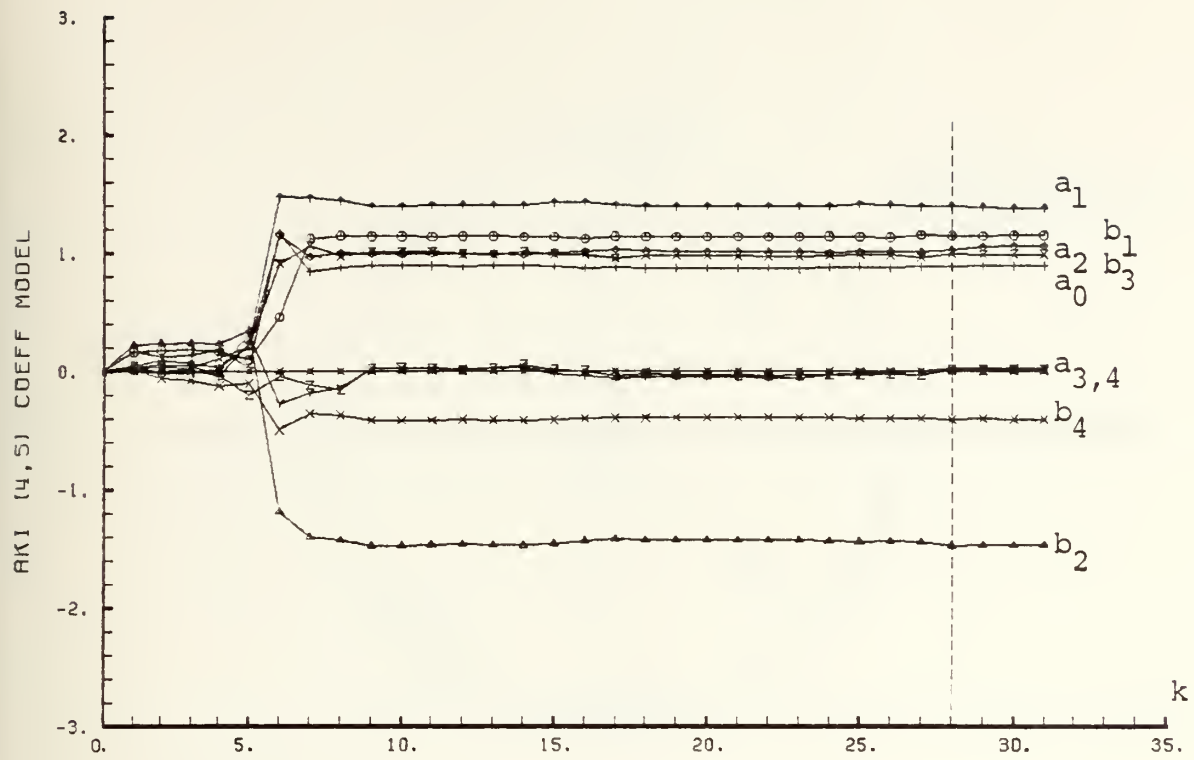


Figure 8.12. AKI(4,5) Computer Coefficients, Perry's Example

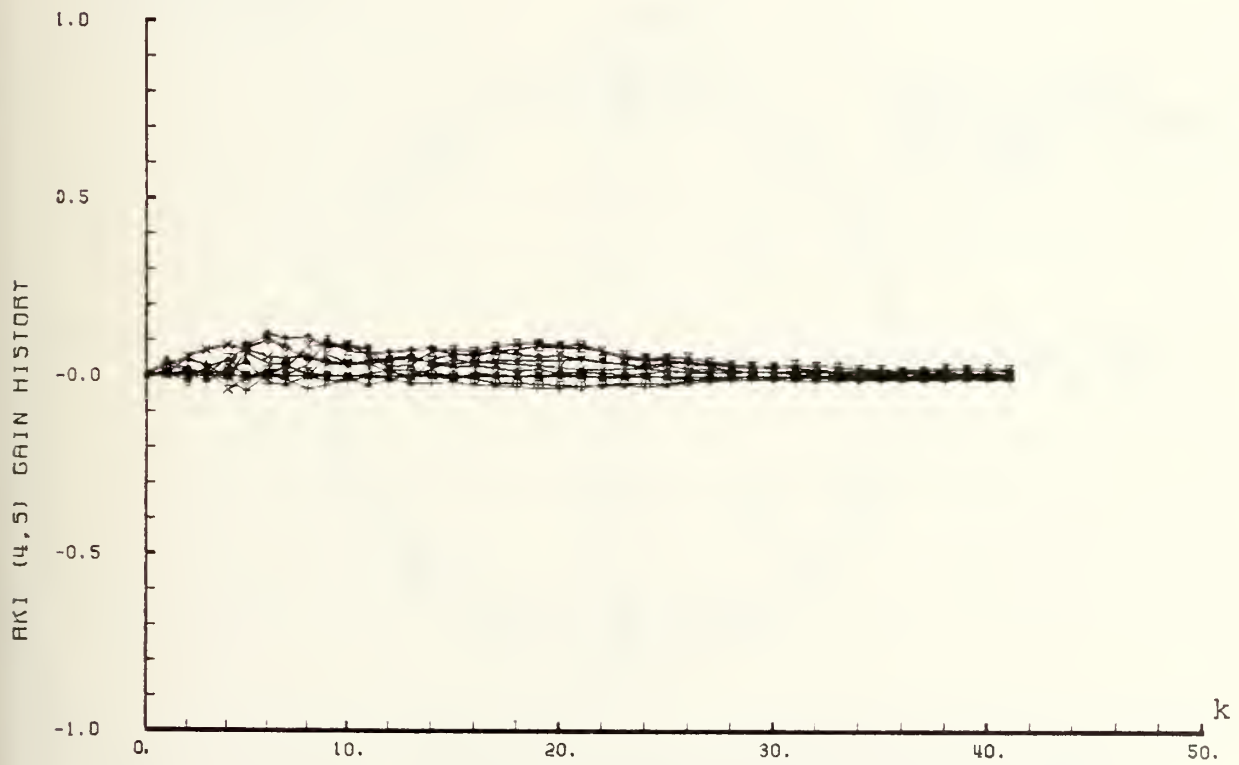
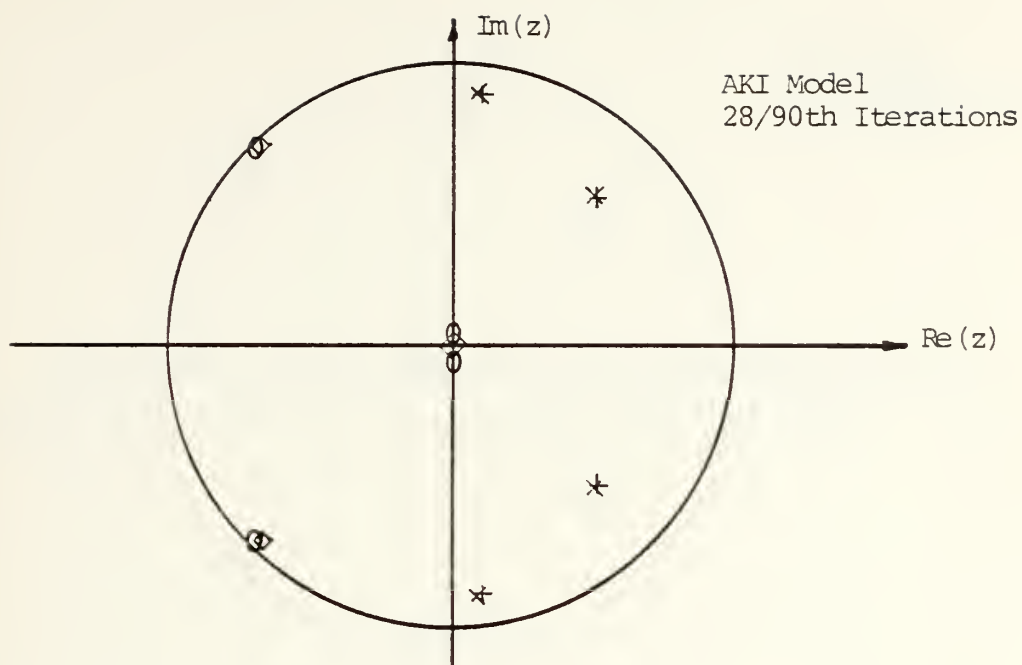


Figure 8.13. AKI(4,5) Averaged Gain Values





X Model Pole  
O Model Zero  
+ System Pole  
◊ System Zero

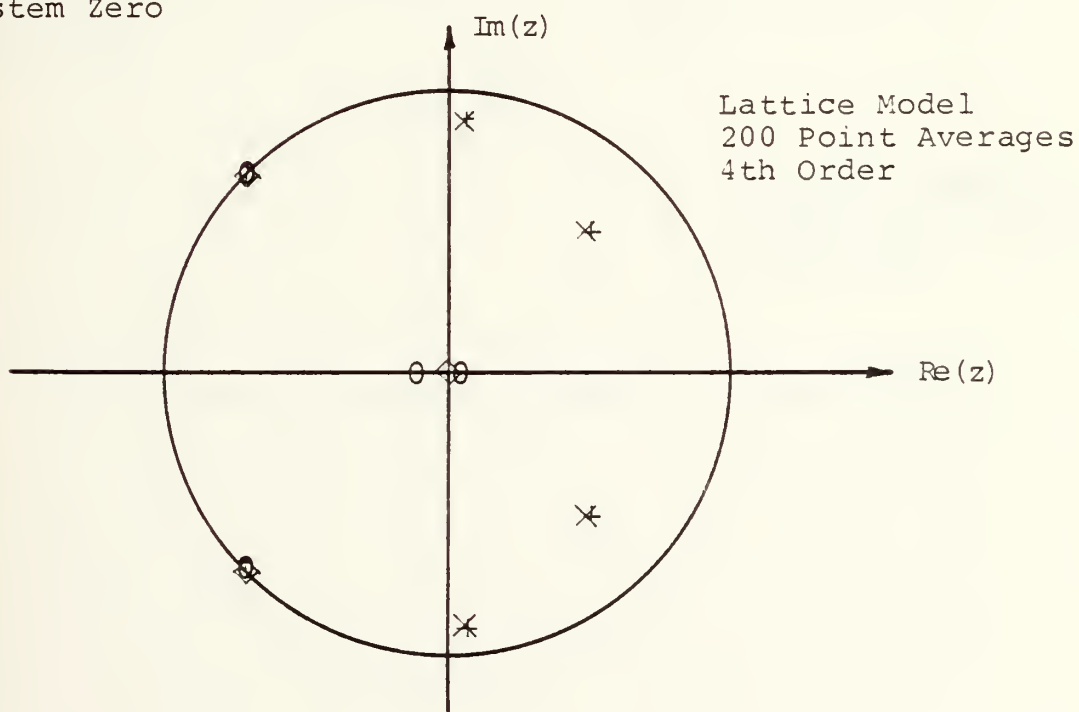
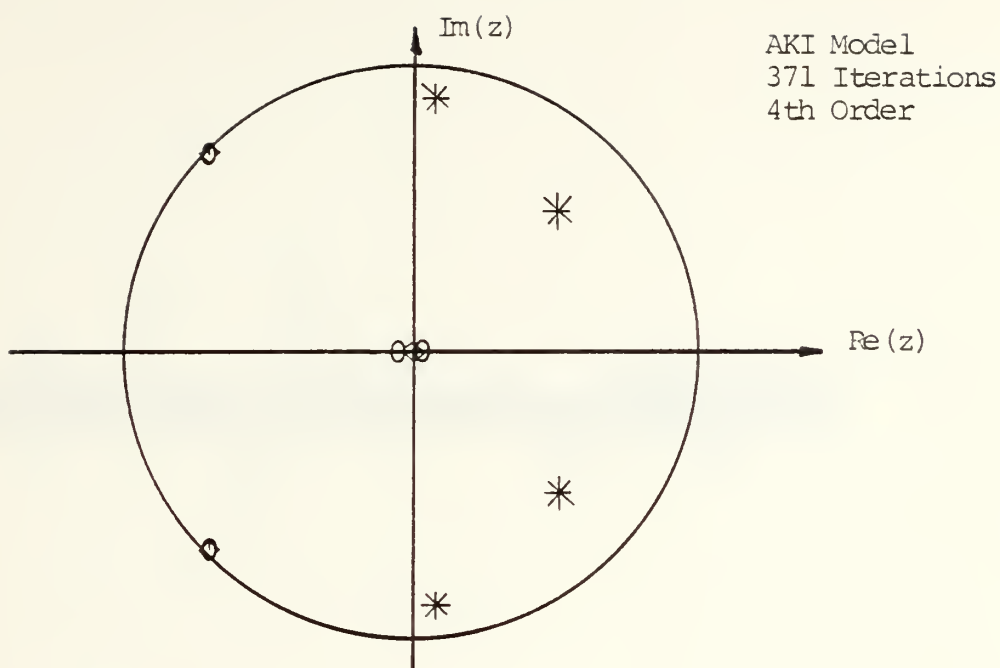


Figure 8.14a. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4)







x Model Pole  
o Model Zero  
+ System Pole  
◇ System Zero

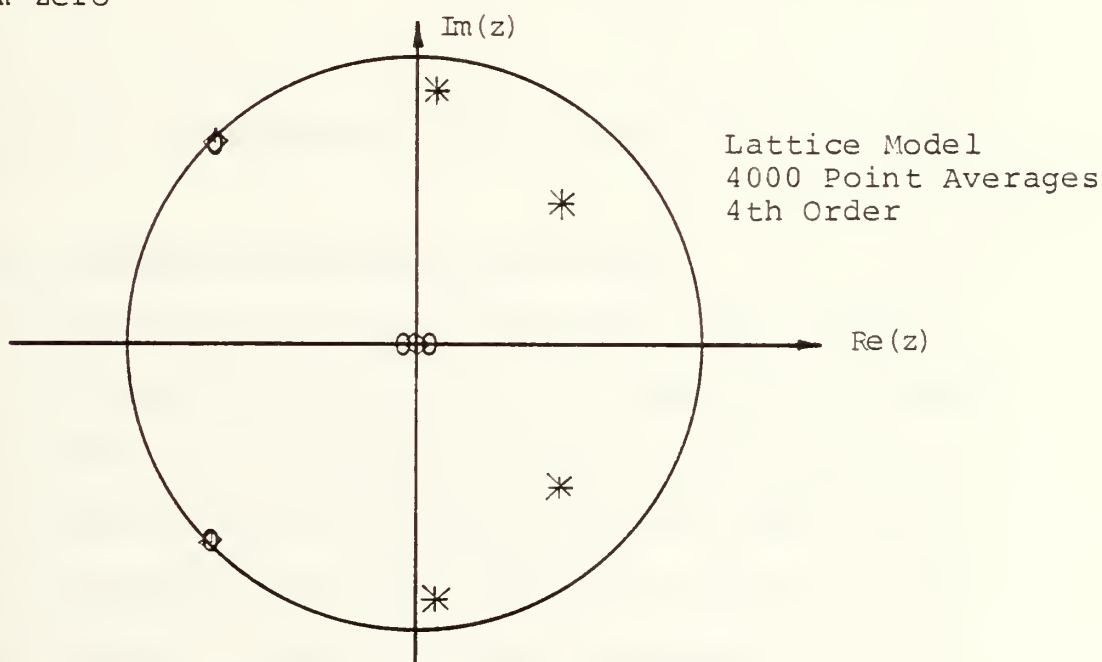


Figure 8.14b. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4) at 371st Iteration



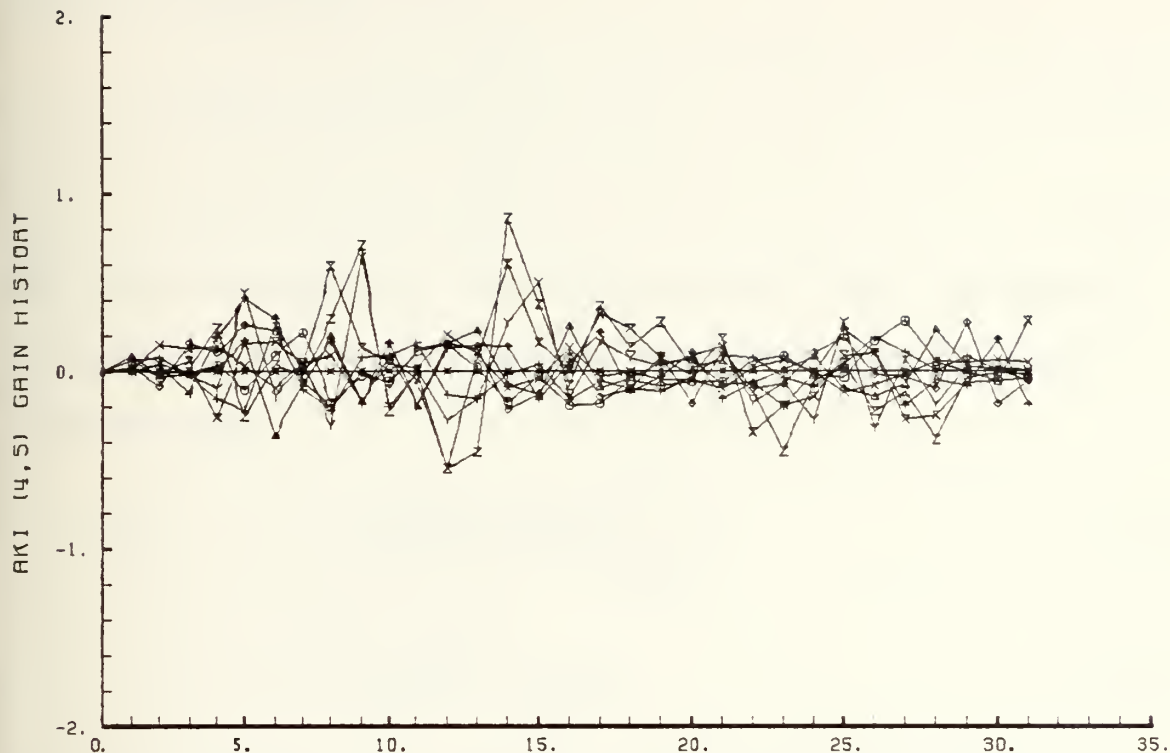


Figure 8.15. Instantaneous Gain Values, AKI(4,5) Model

#### 4. AKI vs Adaptive Recursive LMS Filter

Using Feintuch's proposed algorithm [Ref. 18] and repeating the simulation presented in the rebuttal to Johnson and Larimore [Ref. 19], a comparison was made between the operation of the AKI and the Adaptive Recursive LMS filter. The Adaptive Recursive LMS filter required "tuning" of the convergence factors,  $k_a$  and  $k_b$ , to what appeared to be the optimal performance features of (1) fast convergence and



(2) stable estimates. The "tuning" process was a trial and error procedure beginning with the convergence constants given by Feintuch,

$$k_a = -4.3 \times 10^{-4} \quad (8.5a)$$

$$k_b = -4.3 \times 10^{-4} . \quad (8.5b)$$

The Adaptive Recursive LMS algorithm was then implemented to estimate the coefficients of the plant whose transfer function was,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \quad (8.6a)$$

$$= \frac{a_0 + a_1z^{-1}}{1 - b_1z^{-1} - b_2z^{-2}} \quad (8.6b)$$

The best response obtained by trial and error resulted in the convergence constants,

$$k_a = -4.3 \times 10^{-3} \quad (8.7a)$$

$$k_b = -14.3 \times 10^{-3} . \quad (8.7b)$$

Feintuch reported that after  $8,192^2$  iterations the estimates had converged on the coefficients of (8.6a) with a .2096 normalized rms error. Using the convergence constants (8.7) convergence was essentially reached by the  $4,000^{\text{th}}$  iteration.

---

<sup>2</sup>Feintuch reports the resulting estimates for the coefficients at several intermediate iterations from 8,192 to 65,536, however 8,192 was the minimum number of iterations reported.



It was felt that the operation of the adequately tuned Adaptive Recursive LMS filter could be fairly compared with the operation of the AKI.

Table 8.3 tabulates the performance of the two system identification methods and Figures 8.16 and 8.17 graphically present the responses. Figure 8.18 not only shows at what point the gains of the AKI reach a constant value but also

Table 8.3

ACTUAL VS AKI AND ADAPTIVE RECURSIVE LMS FILTER  
ESTIMATES FOR COEFFICIENTS OF EQUATION 8.6

	ACTUAL	AKI ESTIMATES (k = 150)	ADAPTIVE RECURSIVE LMS (k = 3,990)
$a_0$	0.05	0.05107	0.05045
$a_1$	-0.40	-0.4007	-0.3990
$b_1$	1.1314	1.13200	1.12800
$b_2$	-0.25	-0.25090	-0.2442

provides a measure of confidence that the unknown system has been identified. It is evident from Figure 8.19 that both methods seem to identify the poles and zeros of the actual plant; however, the poles computed by the AKI are closer. The zeros computed by the AKI are approximately .080 units farther from the true zero than is the Adaptive Recursive LMS filter estimate. All aspects considered, the AKI takes considerably fewer iterations to arrive at its estimate.





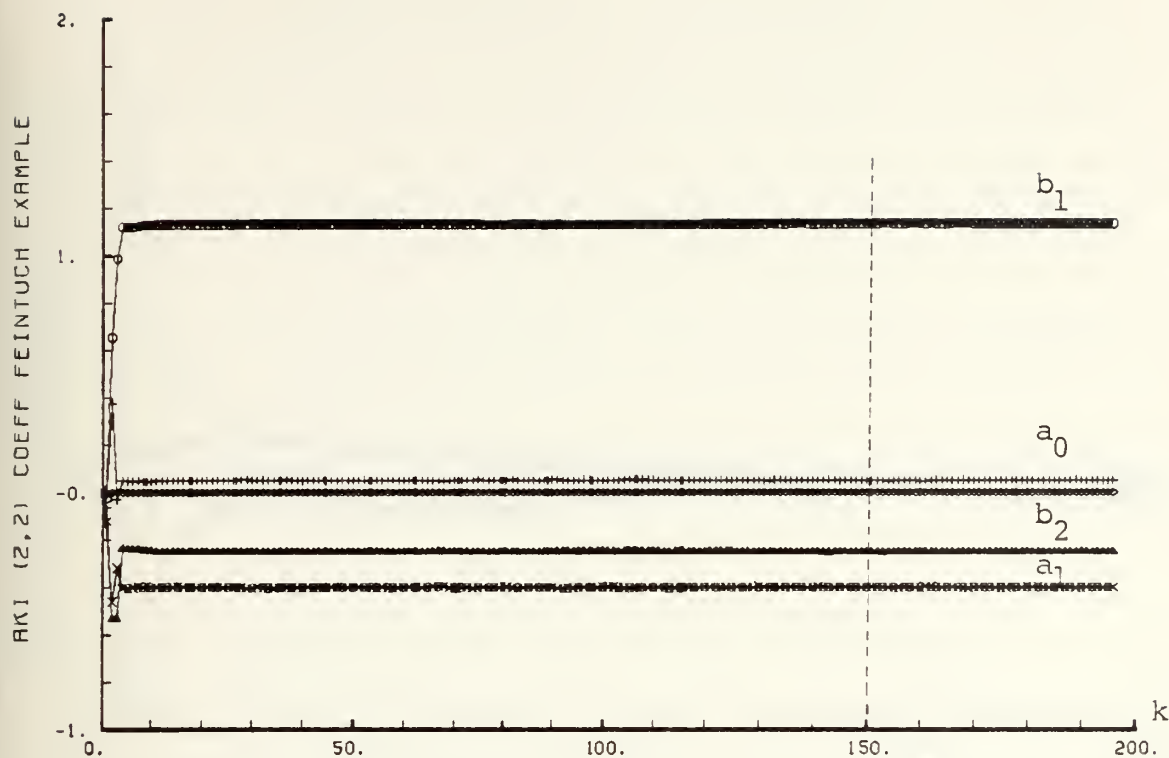


Figure 8.16. AKI(2,2) Computed Coefficients, Feintuch Example

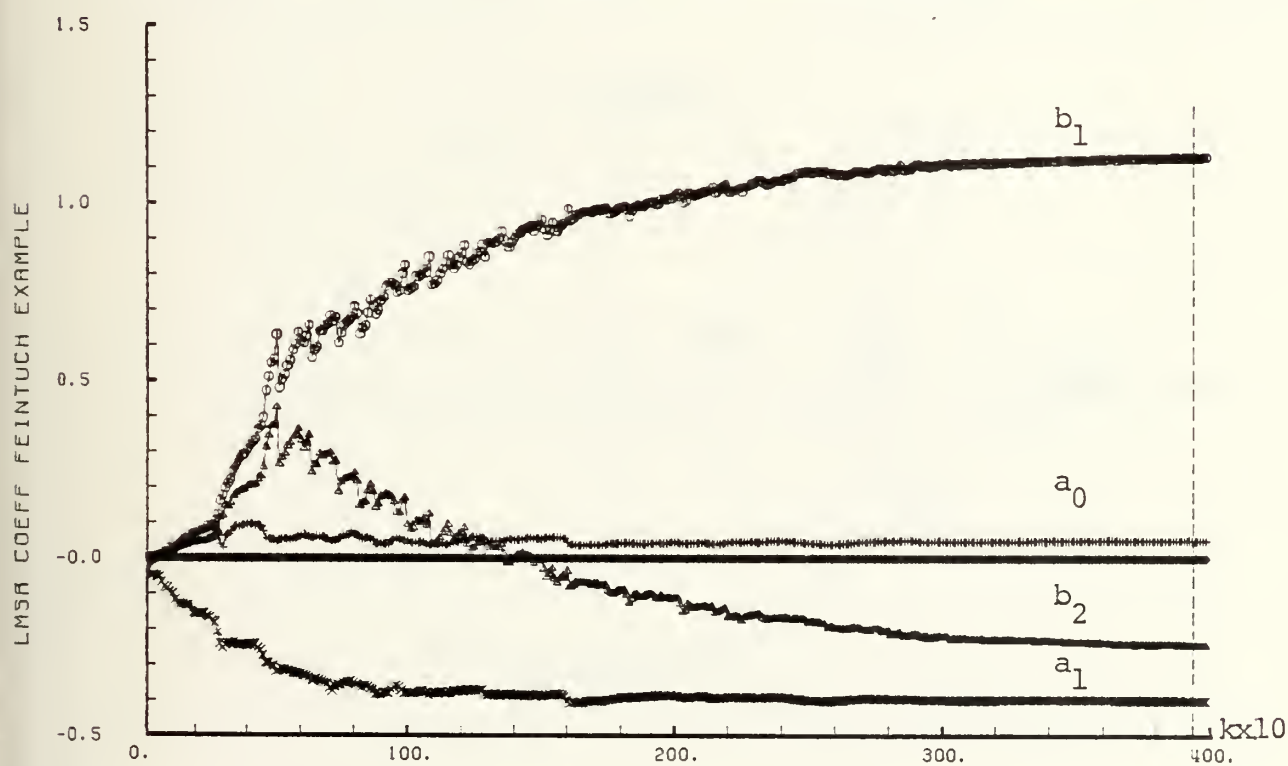


Figure 8.17. LMSR Computed Coefficients, Feintuch Example



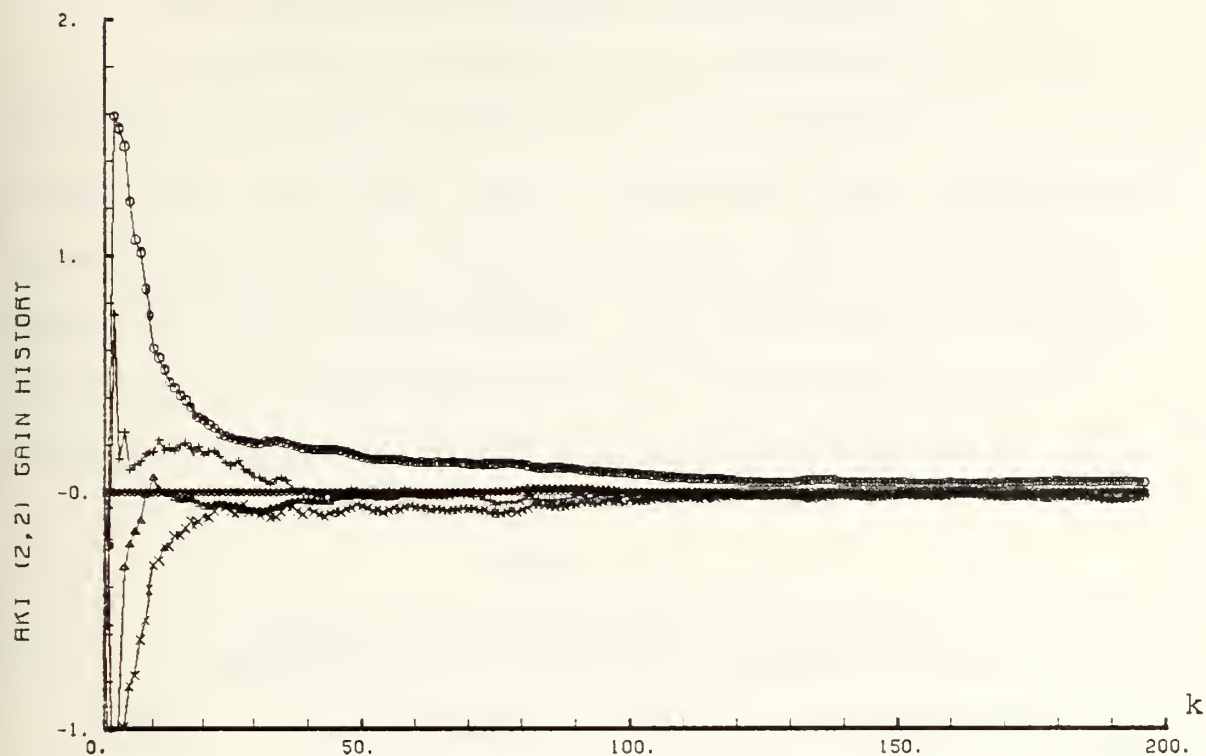


Figure 8.18. AKI(2,2) Averaged Gains, Feintuch Example

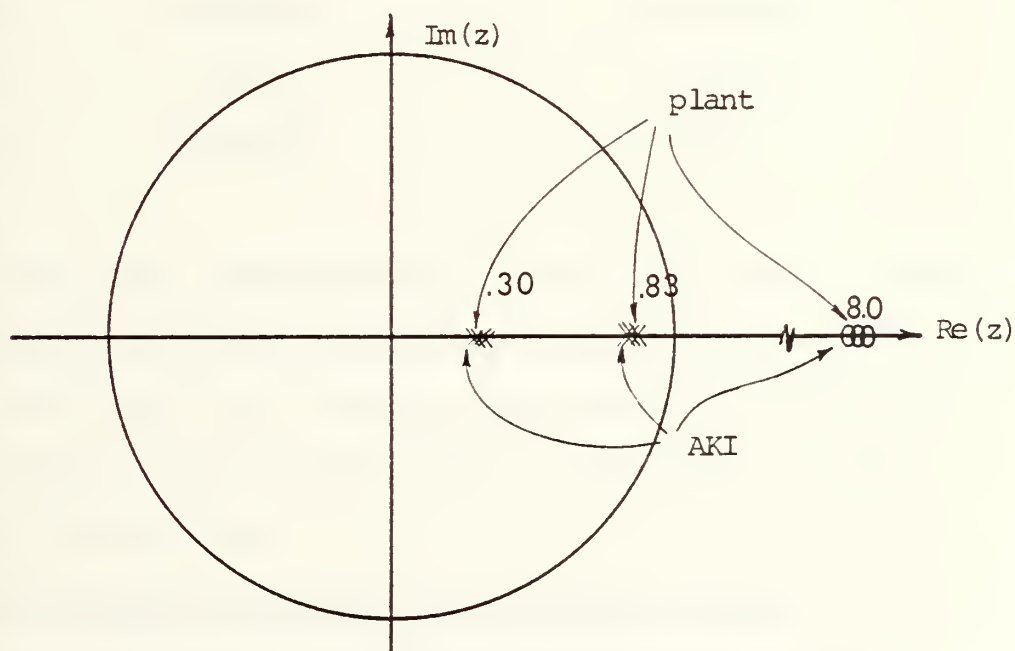


Figure 8.19. Pole/Zero Models Produced by AKI and Adapt. Rec. LMS Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.6)



## 5. AKI Applied to PLL Data (Linear Region)

To identify the coefficients associated with the general autoregressive-moving average representation for the phase locked loop, the input noise signal power (mean square value) was kept at  $(10 \text{ deg})^2$ . In this manner, the linear region of the PLL was invoked. The input/output data was analyzed by the AKI resulting in the estimates shown in Table 8.4. Figures 8.20 and 8.21 show the response of the AKI

Table 8.4

ACTUAL VS AKI ESTIMATES OF THE ARMA  
REPRESENTATION FOR THE PLL (LINEAR REGION)

	ACTUAL	AKI ESTIMATES (k = 82)	AKI ESTIMATES (k = 350)
$a_0$	0.000000	-0.0001046	-0.000692
$a_1$	0.020000	0.0198500	0.0199800
$a_2$	-0.018890	-0.018720	-0.0188900
$b_1$	1.980000	1.980000	1.978000
$b_2$	-0.981110	-0.981000	-0.9790

when applied to the identification of the PLL data. It was noted that though the AKI correctly identified the coefficients of the linear PLL, the AKI gains were large due to the weak signals (input/output data) incorporated in the measurement vector,  $H(k)$ .

## 6. AKI Applied to PLL Data (Non-linear Region)

When analyzing any non-linear system the engineer must bring to bear all his analysis techniques on the problem. The PLL was therefore studied using classical root locus



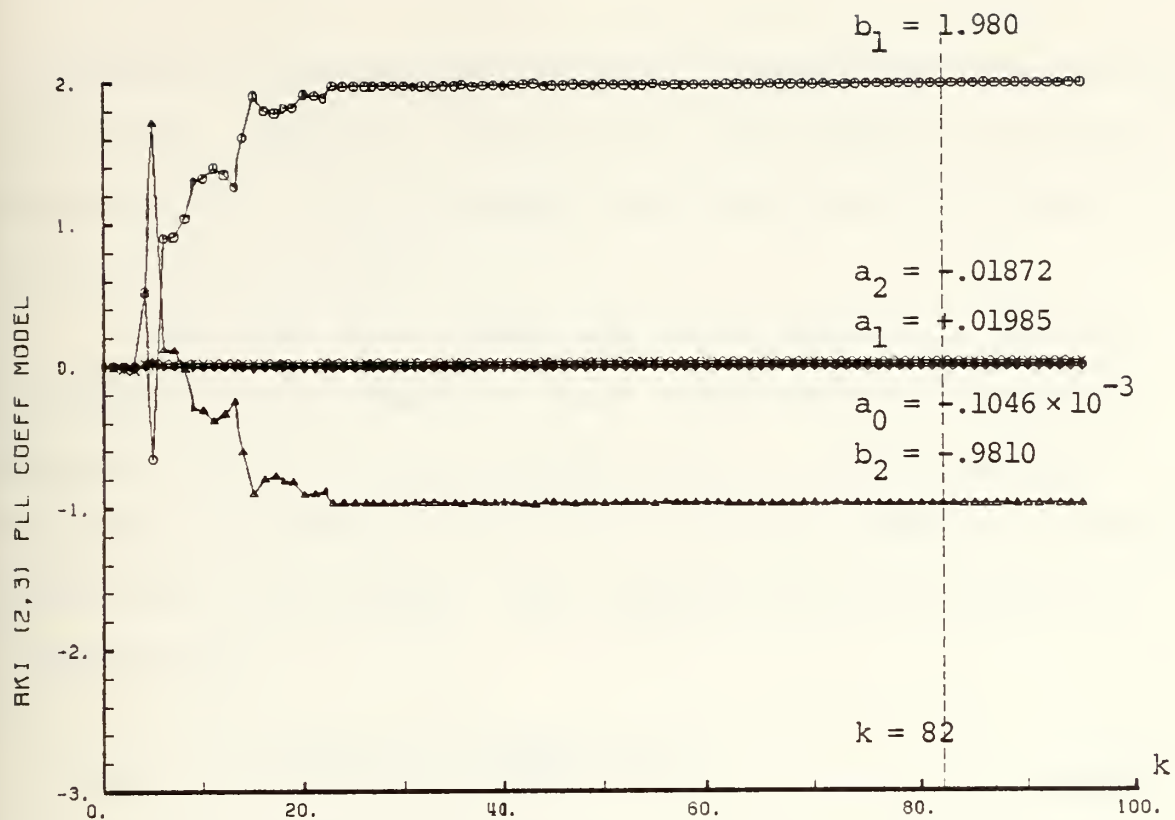


Figure 8.20. PLL, AKI(2,3) Computed Coefficients

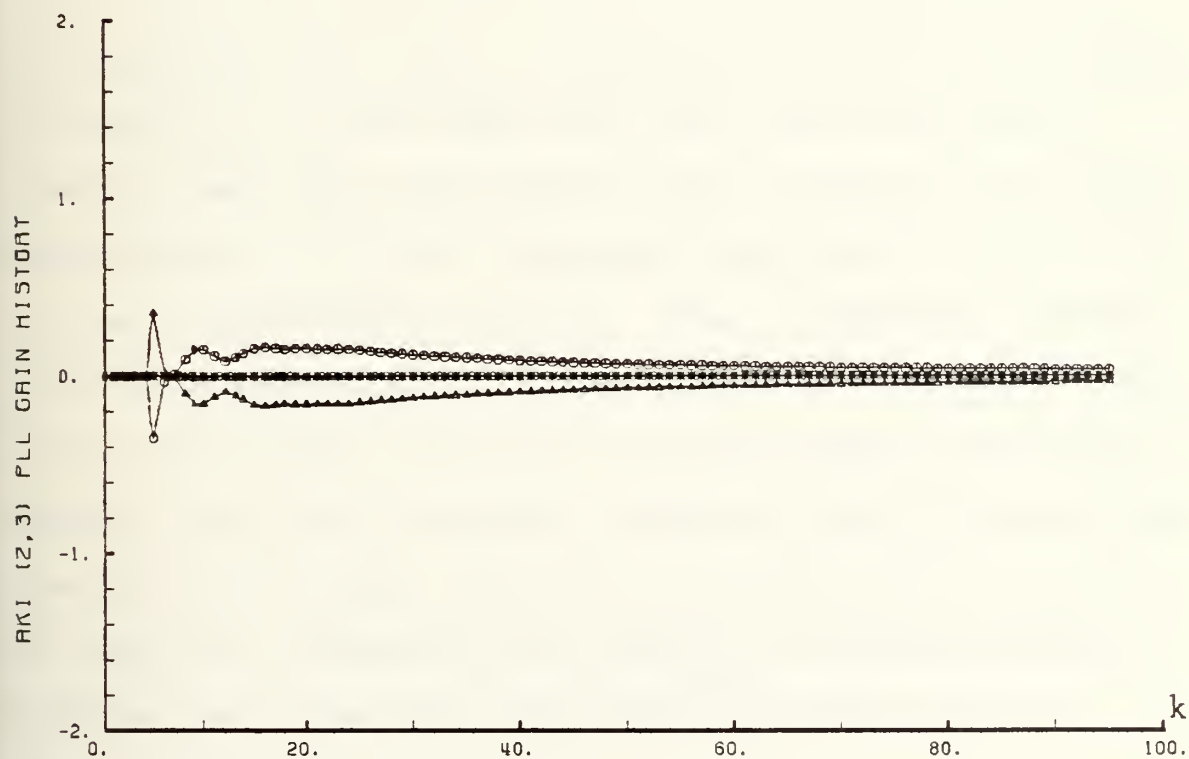


Figure 8.21. Averaged Gain Values, AKI(2,3) PLL Model





techniques to preview the possible outcomes when being modeled by the AKI. Stability analysis of a more complex PLL system using the root locus technique has been previously presented in the literature [Ref. 40].

The root locus technique can be applied to the PLL presented in this thesis by first assuming that the  $\sin(\cdot)$  block of Figure 5.7 is a variable gain,  $\lambda$ . This is not a restrictive assumption since the PLL during operation generally tracks small deviations. The loop gain can be written by inspection as,

$$\begin{aligned} L(z) &= \frac{\lambda(.02)(1 - .9445z^{-1})z^{-1}}{(1 - z^{-1})^2} \\ &= \frac{\lambda(.02)(z - .9445)z}{(z - 1)^2} \end{aligned} \quad (8.8)$$

From equation (8.8) the root locus of  $L(z)$  is drawn as shown in Figure 8.22. Even though the root locus technique is generally used when the signals in the system are considered deterministic, it is not surprising that some of the results obtained are nevertheless valid. When a moderately strong input noise identification signal  $[E\{u^2(u)\} < (25 \text{ deg})^2]$  was used, the pole-zero locations of the PLL seem to follow the classical root locus analysis. However, when the input noise identification signal power is increased beyond  $(25 \text{ deg})^2$  the pole-zero locations do not follow the expected behavior predicted using the root locus method as can be seen in Figure 8.22.



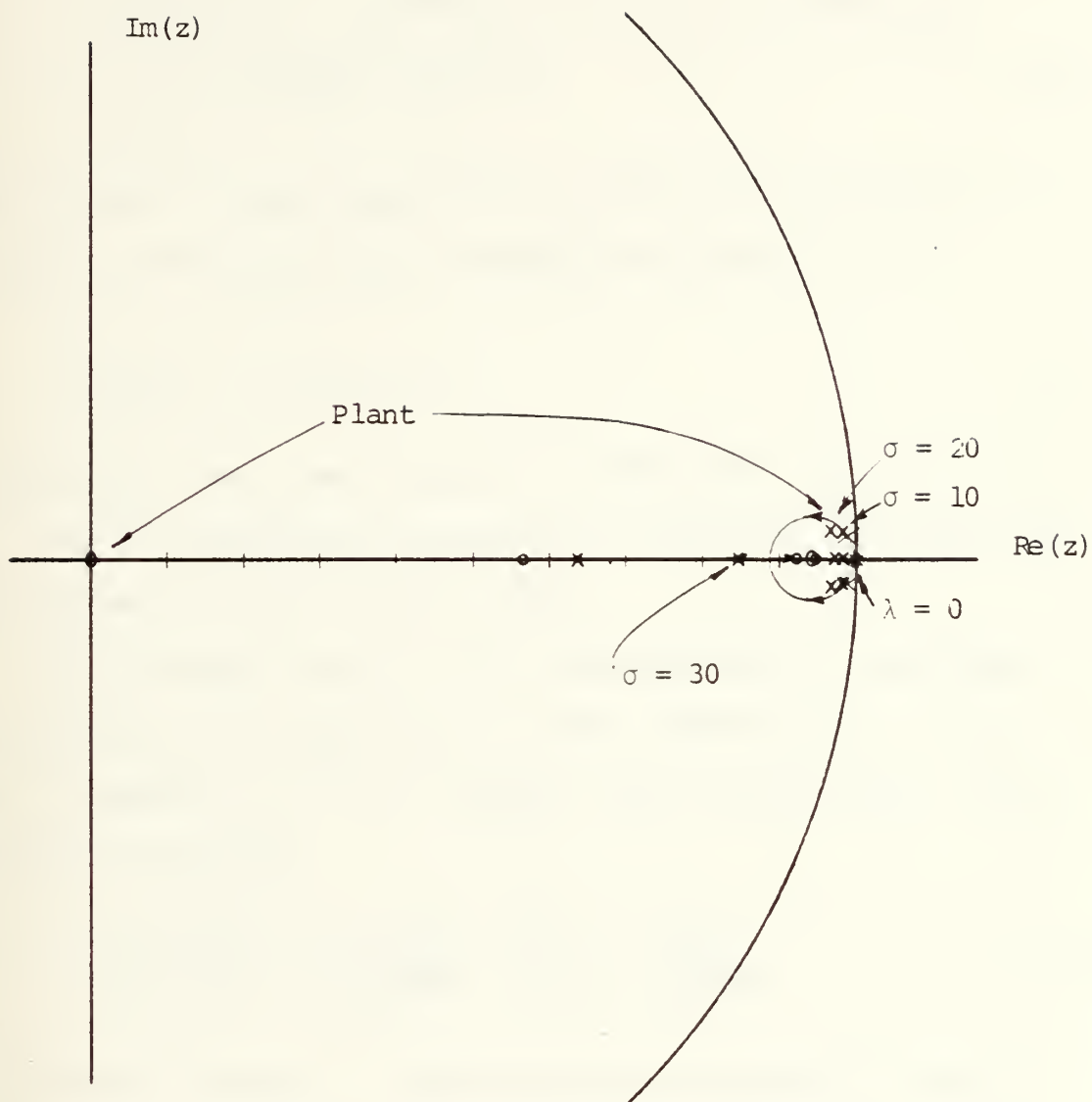


Figure 8.22. PLL Root Locus Analysis vs Computer Roots of the AKI ARMA Model



The departure of the pole-zero behavior from what was expected was analyzed by closer investigation of the linear terms of equation (7.8).

$$\begin{aligned} y(k)_{\text{lin}} = & .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\ & + (.01889\alpha - 1)y(k-2) \end{aligned} \quad (8.9)$$

Recalling that a third order Taylor series approximation of the sine is the functional expressed by equation (7.7),

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.7)$$

one notes that the linear region is described when  $\alpha = 1$  and  $\beta = 0$ . It is therefore reasonable to study the variation of  $\alpha$  with respect to input noise power.

The average value  $\bar{\alpha}$  of  $\alpha$ , was computed by equating the estimated AKI coefficients to the coefficients of like terms in equation (8.9) for several input noise power levels. The relation used was

$$\bar{\alpha} = \frac{1}{4} \left[ \frac{2.0 - \hat{b}_1}{.02} + \frac{1.0 + \hat{b}_2}{.01889} + \frac{\hat{a}_1}{.02} - \frac{\hat{a}_2}{.01889} \right] \quad (8.10)$$

at the 350th iteration. The relationship between  $\bar{\alpha}$  and the input noise power level is readily apparent from Figure 8.23. This result is plausible since if one considers the input-output relationship of the sine block of Figure 5.7 one obtains Figure 8.24. Superimposing the gaussian probability functions of the different input noise signals, it can be seen that for



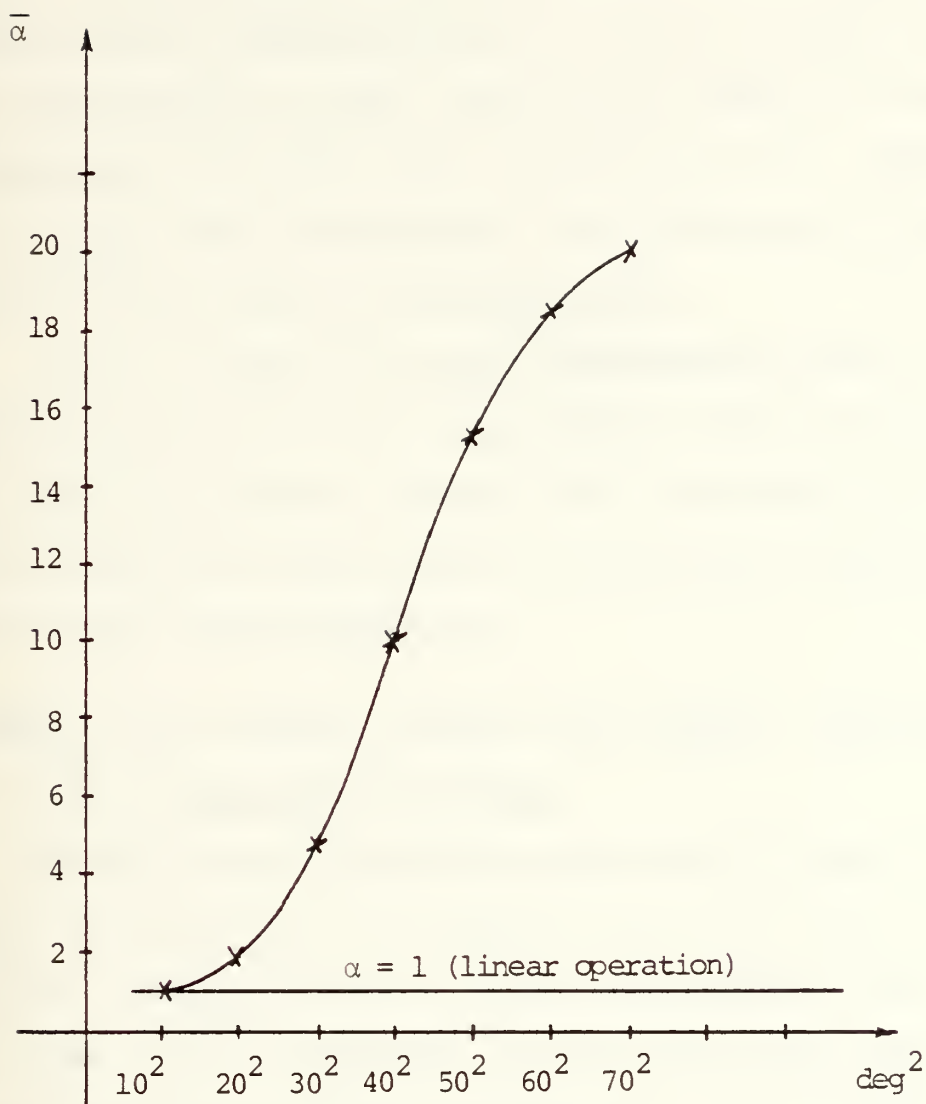


Figure 8.23. Variation of  $\bar{\alpha}$  vs Input Noise Signal Power





low power levels of the input signal, identification of the linear parameters of the overall system (equation 5.9) can be made. As the input power is increased beyond  $(25 \text{ deg})^2$  the input-output characteristics of the  $\sin x$  block are no longer approximately linear, showing its effect in Figure 8.24 as a departure from its linear operation,  $\bar{\alpha} = 1$ . Therefore, by monitoring  $\bar{\alpha}$  one can determine when the overall system is entering its non-linear operating regime.

Since the same functional dependence between each of the AKI estimates and  $\bar{\alpha}$  did not exist for all of the input noise powers considered, knowing  $\bar{\alpha}$  did not provide any information of the pole locations but did provide a measure of the degree of non-linear operation.

## B. ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING)

### 1. AKI vs LMS Adaptive Filter

Using the data derived by operating a plant with the transfer characteristic of equation (8.1) the operation of the AKI and the LMS adaptive filter was compared when the orders used in the identifier and the LMS filter were greater than the known process that generated it. The following overmodeling cases were studied:

- (1) MA model is greater than plant MA process
- (2) ARMA model is fitted to MA plant

When the MA model order is greater than that of the plant MA process, it was found that both the AKI and the LMS filter would compute coefficients close to zero for the higher



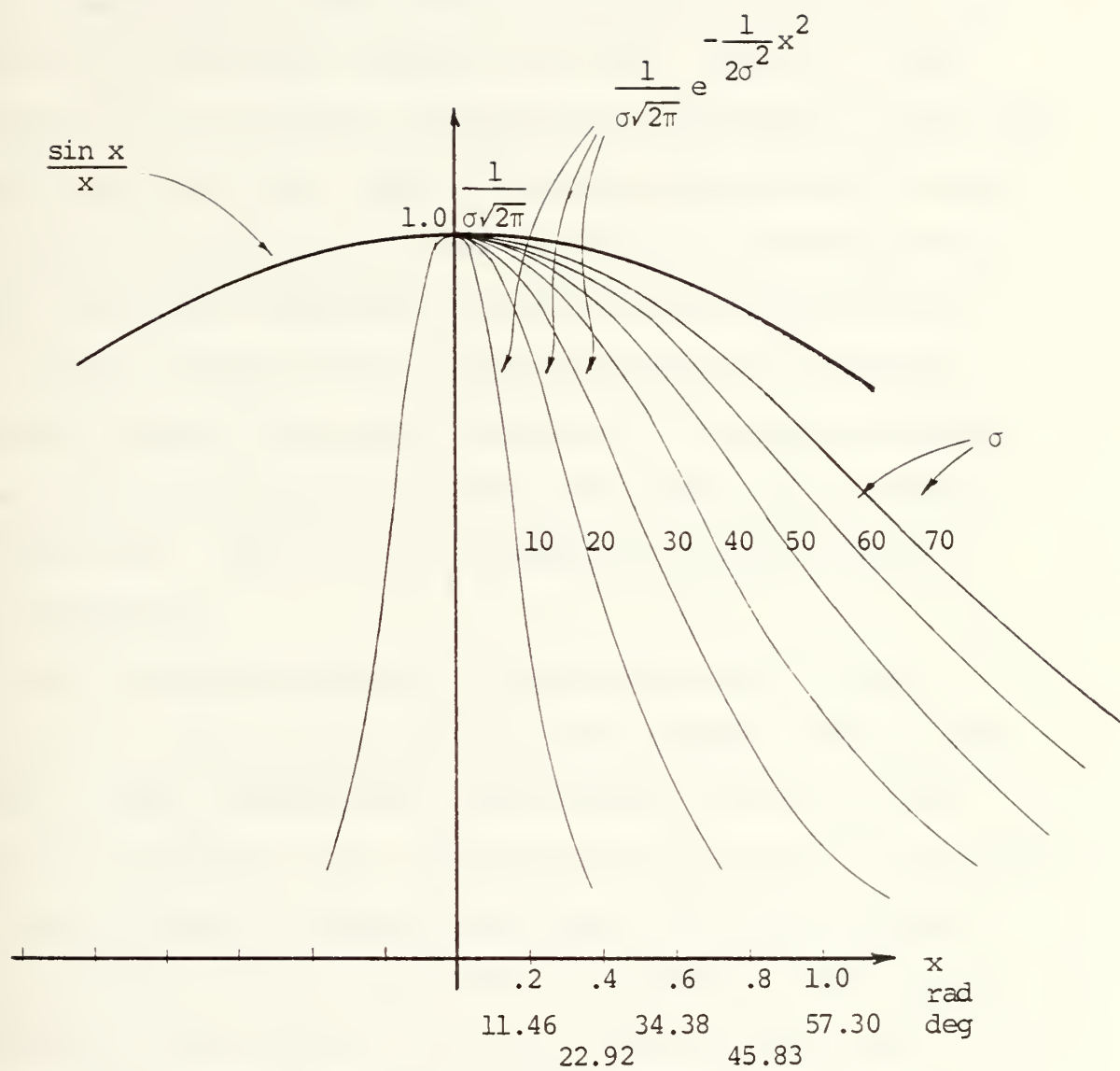


Figure 8.24. Comparison of  $\frac{\sin x}{x}$  and p.d.f. of input noise signals



order coefficients. This effect can be seen in Figures 8.25a and 8.25b when a fourth order MA process is used to model the actual second order process represented by equation (8.1). Since these are a "zeros only" plant and model the overmodeling essentially causes zeros to appear at the origin of the Z-plane of the model transfer function. Table 8.5 summarizes the results for two overmodeling conditions at the 250th iteration using the AKI. When a purely moving average process represented by equation (8.1) was modeled as an ARMA process, one must direct his attention to the poles and zeros of the model transfer function which the AKI computed and compare them to the actual plant poles and zeros. It was not readily apparent from the resulting coefficients that the plant had been identified. The following example will help clarify what is happening.

For the data produced by the second order plant (equation 8.1), an autoregressive moving average (ARMA) model of orders 2 and 5 respectively was fitted. It can be seen from Table 8.6 that no firm conclusions can be drawn about which coefficients actually identify the plant. We, the analysts, knowing the form of the plant which produced the data, could qualitatively state that  $b_1$ ,  $b_2$ ,  $a_3$ ,  $a_4$  and  $a_5$  are small enough to be ignored. Hence, we can identify the plant correctly. However, inspection of the poles and zeros of the transfer function which the AKI computed,



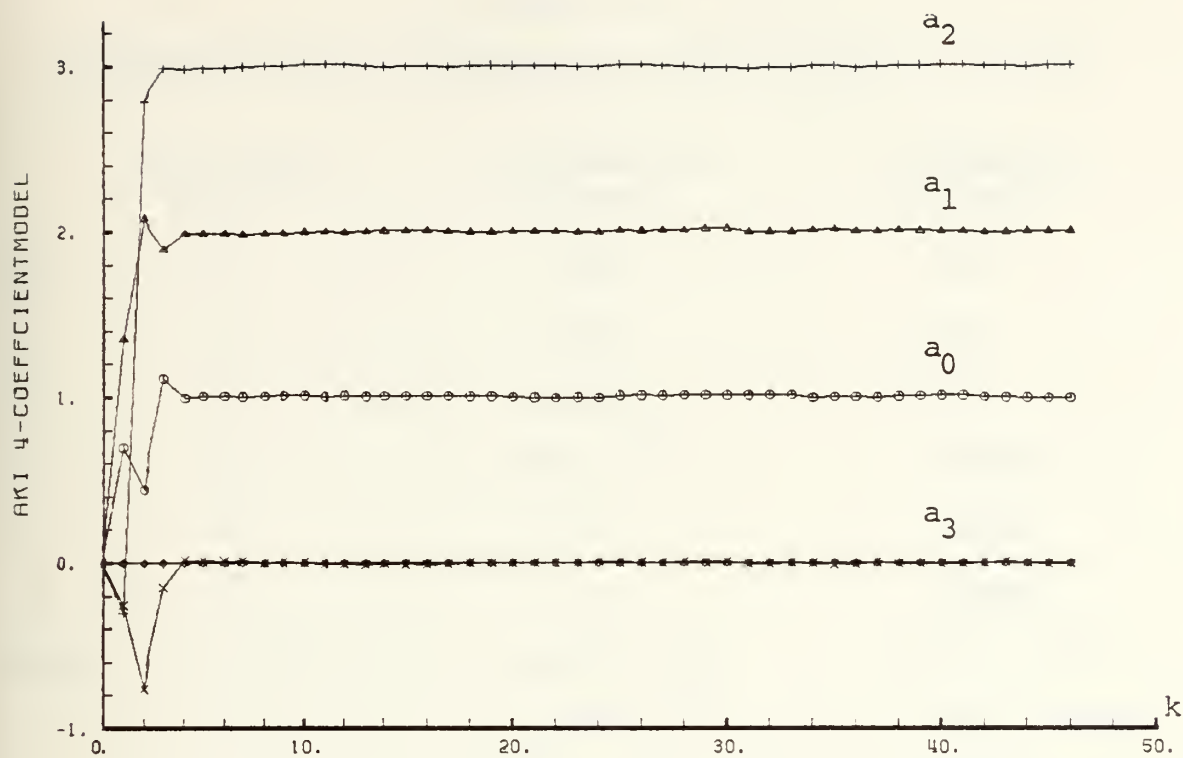


Figure 8.25a. AKI(0,4) Model of a Third Order MA Process

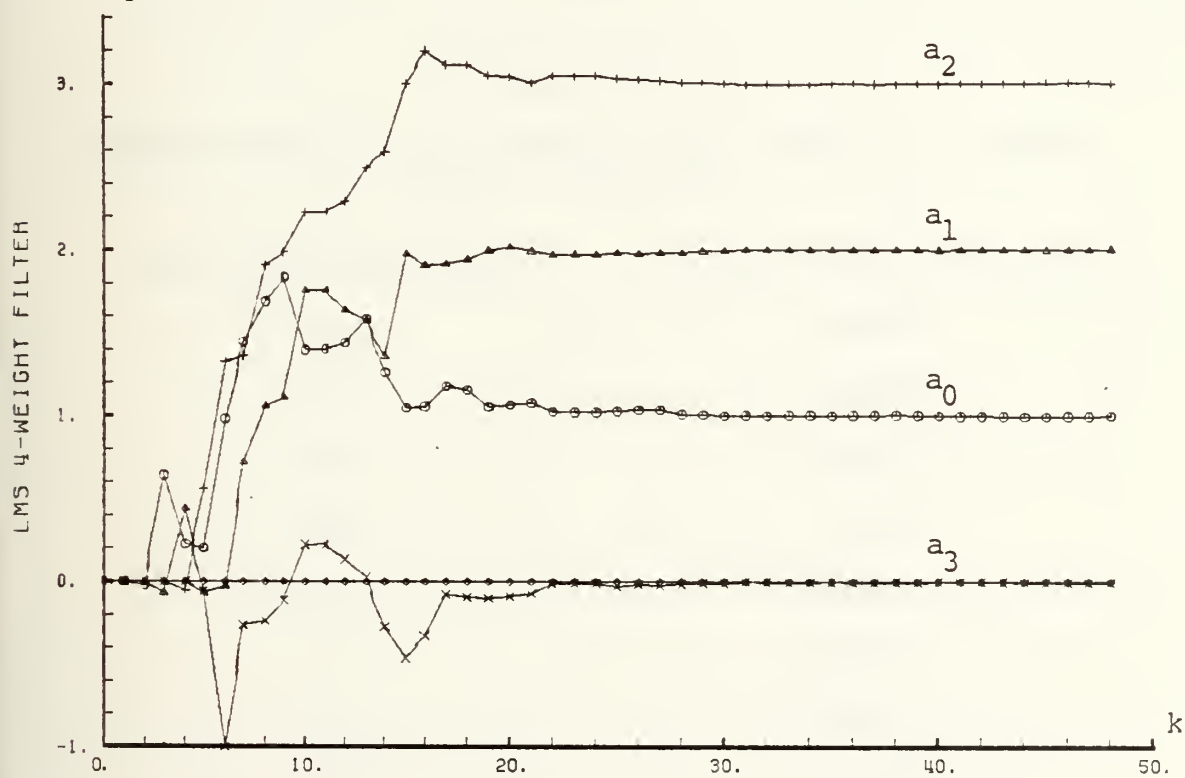


Figure 8.25b. Fourth Order LMS Filter Weights of a Third Order MA Process





Table 8.5

OVERMODELING OF A MA PROCESS USING THE AKI ( $k = 250$ )

	Actual Model Coeffs	3rd Order AKI MA Model	5th Order AKI MA Model
$a_0$	1.0	.9995	.9991
$a_1$	2.0	2.0000	2.000
$a_2$	3.0	3.0000	3.000
$a_3$	0.0	$.893 \times 10^{-3}$	.0003191
$a_4$	0.0	---	.001082
$a_5$	0.0	---	.0003198
zeros:	$-1.0 \pm 1.414j$	$-1.000 \pm 1.414j$	$-1.001 \pm 1.414j$
		$-.2977 \times 10^{-3}$	$.0227 \pm .0428j$
			-.4534

Table 8.6

OVERMODELING OF A MA PROCESS USING THE AKI OF ORDERS  
AR = 2, MA = 6

	Actual Plant Coefficients	AKI(2,6) Model Coefficients
$b_1$	0.0	.00467
$b_2$	0.0	.10500
$a_0$	1.0	1.0000
$a_1$	2.0	1.9940
$a_2$	3.0	2.8840
$a_3$	0.0	-0.2257
$a_4$	0.0	-0.3142
$a_5$	0.0	0.0001517



$$H_{AKI(2,6)}(z) = \frac{1.0 + 1.994z^{-1} + 2.8840z^{-2} - 0.2257z^{-3} - 0.3142z^{-4} + 0.001517z^{-5}}{1 - .00467z^{-1} - .1050z^{-2}} \quad (8.11)$$

results in:

poles:   -.8217  
           .3264  
 zeros:   -.9996 ± 1.414j  
           .4827 × 10<sup>-3</sup>  
           -.3212  
           .3262

The observation to be made is that there are two pole-zero combinations which are near cancellation. This suggests that the AKI algorithm be rerun with the AR and MA orders reduced by at least two.

The implication of the analysis of this section is that a model can in theory be found for a given set of input/output data. Further, a parsimonious model can be identified by careful observation of pole-zero combinations which are near cancellation and of stray zeros near the origin.

## 2. AKI Applied to AR and ARMA Data

Essentially the same characteristic results found in Section VIII.B.1 were confirmed when the data produced by plants defined by the transfer functions of equations (8.3) and (8.4) were analyzed by the overmodeled AKI. That is, pole-zero pairs near cancellation and zeros near the origin



were produced by the AKI. Figures 8.26 through 8.28 have been chosen as representative pole-zero plots of the transfer functions computed by the AKI for the AR plant of Section VIII.A.2 and for Perry's model (Section VIII.A.3).

### C. CONCLUSION

This work indicates that the Kalman filter algorithm heretofore generally used as a state estimator, or in augmented form to estimate parameters (in which case the parameters are treated as states), can also be formulated in an adaptive manner to iteratively estimate the coefficients of an ARMA equation explicitly. This approach, termed the Adaptive Kalman Identifier (AKI), summarily identifies the unknown system whose input/output data is being processed. The LMS adaptive algorithm of Widrow, and its modification by Griffiths (in which the convergence factor is selected to be inversely proportional to the input signal power) are shown to be sub-optimal cases of the AKI. An additional insight provided by the AKI is that it indicates clearly how measurement noise might be taken into account in the LMS adaptive formulation.

The operation of the AKI was checked by way of simulation and compared with two existing identification techniques: (1) the LMS Adaptive nonrecursive (MA) algorithm and (2) the Adaptive Recursive ARMA LMS algorithm. It was found that not only are the two LMS filtering techniques special suboptimal cases of the AKI; but, further, the AKI exhibits superior convergence and modeling properties for the cases where (1) the



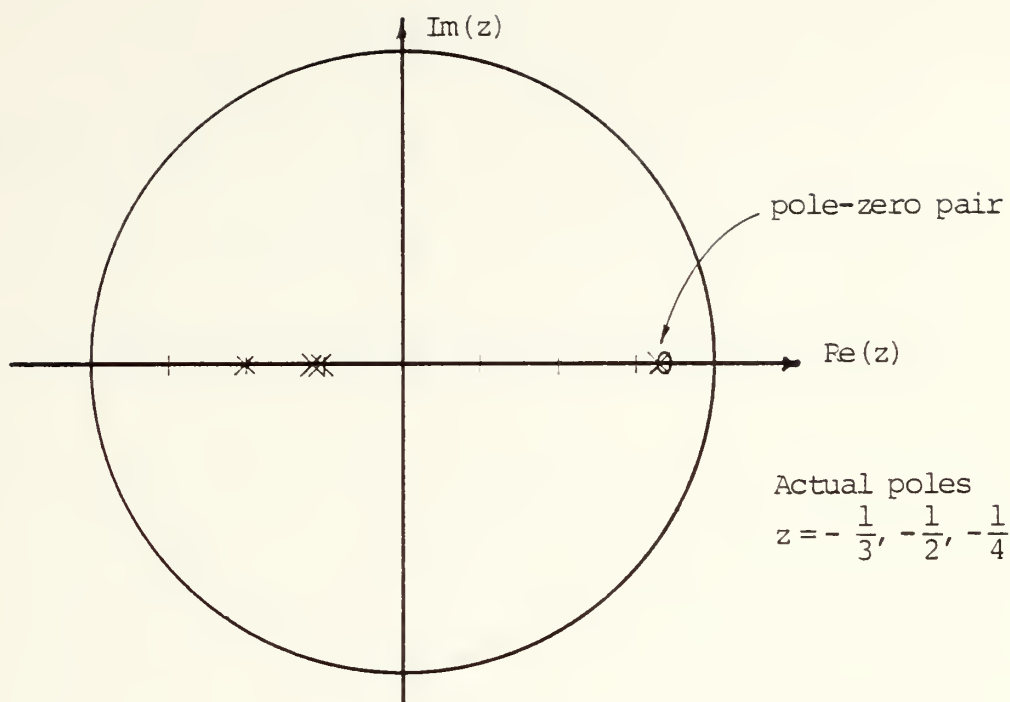


Figure 8.26. AKI(3,1) Overmodeled AKI(4,2)

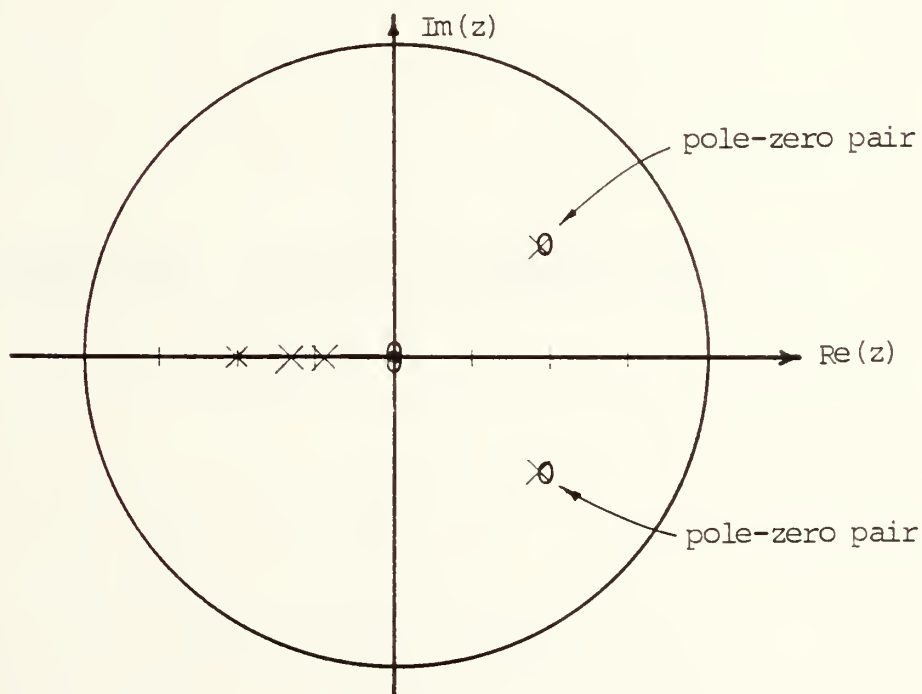


Figure 8.27. AKI(3,1) Overmodeled AKI(5,5)





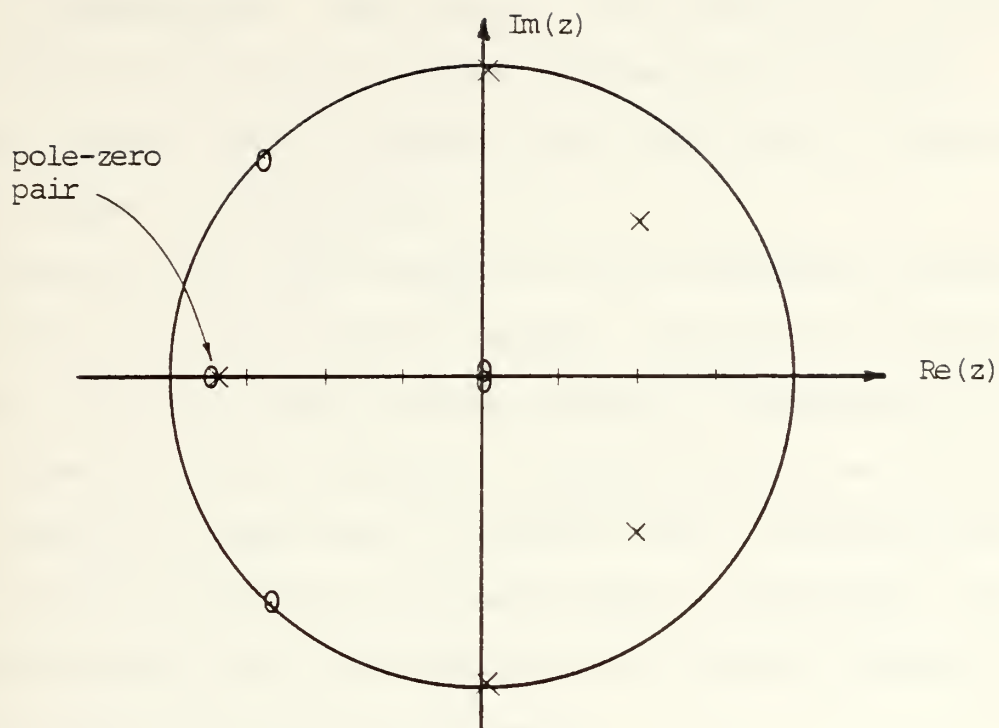


Figure 8.28. AKI(4,5) Overmodeled AKI(5,6)



order of the unknown plant is known and (2) the order of the plant is overmodeled. Additionally, the simulations indicate that accuracies similar to those obtained using lattice modeling methods, can be achieved using the AKI at a decidedly smaller number of iterations.

By making minor modifications to the measurement vector,  $H(k)$ , that is by using hybrid signals, the AKI was used to identify the linear and non-linear ARMA representations of a phase locked loop with success. Interestingly, the AKI technique appears to enable one to discern when a potential non-linear system enters its non-linear mode of operation, by closely monitoring the coefficients of the linear portion of the generalized non-linear ARMA model.

#### D. TOPICS FOR FURTHER CONSIDERATION

Several areas for further study directly and indirectly related to the AKI were uncovered. Foremost, a rigorous convergence proof is desirable. Although the connection was made between the AKI algorithm and the initial equation from which the lattice modeling algorithm is developed, similar comparisons (such as Chapters III and IV) could provide more insight into the operation of both. The multichannel AKI is the logical development of the single channel AKI presented. And, lastly, refinement of the AKI software (using the NL = 5 option) to include ARMA modeling of time series using a Box-Jenkins approach [Ref. 26] is feasible. A limited number of simulations using Monterey rain data and series C [Ref. 26]



data seem to indicate that an application exists for the AKI  
in this area.



## APPENDIX A

### THE DISCRETE WEINER PROBLEM

The discrete Wiener problem considers optimally filtering a desired signal from unwanted stationary noise. The criterion of optimality used is minimization of the mean squared error between the output and the desired signal. Generally one desires that the optimal filter, which is the device being sought, be time invariant and that it be able to accept a signal,  $s(k)$ , and noise,  $n(k)$ , where each are samples from stationary random processes. In other words, we want a device which can accept

$$x(k) = s(k) + n(k) \quad (A.1)$$

as an input and produce at its output  $s(k+\Delta)$  or some linear function thereof where  $\Delta$  is some known delay.

Assuming that the desired output,  $d(k)$ , is the response to a specified sampled data linear system whose transfer function,  $H_d(z)$ , is given, then the error between the desired output and the output of the filter,  $y(k)$ , we seek is,

$$e(k) = d(k) - y(k) . \quad (A.2)$$

Figure A.1 depicts the discrete Wiener problem formulation. The derivation from here follows the one presented by Maybeck [Ref. 27] for the continuous case. From the block diagram we have,





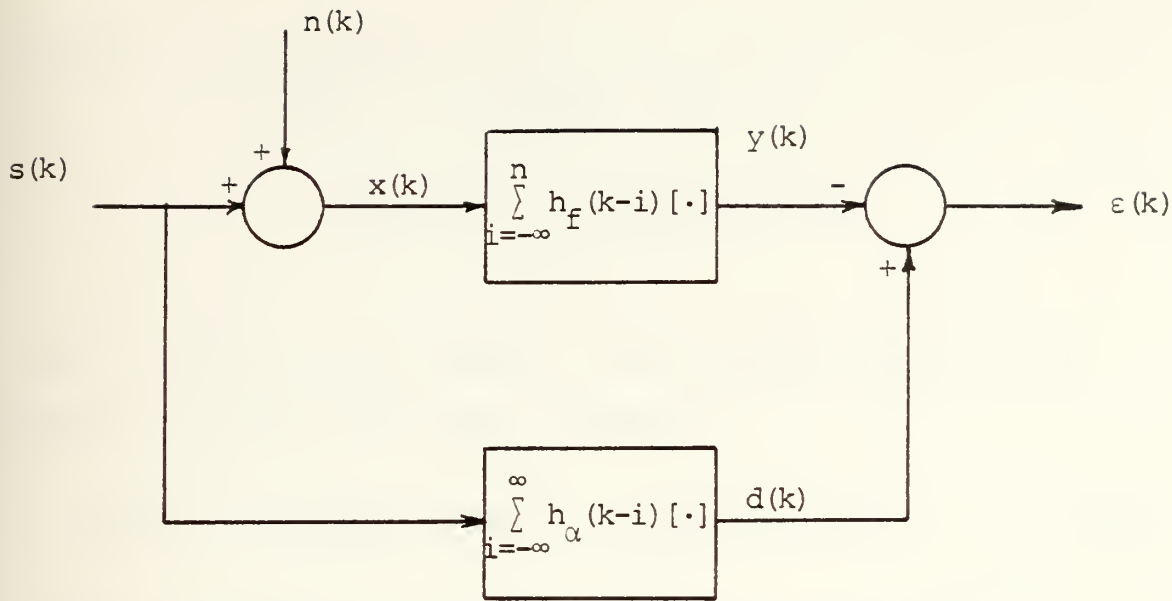


Figure A.1. The Discrete Wiener Problem

$$y(k) = \sum_{i=-\infty}^{\infty} h_f(k-i) x(i) \quad (A.3)$$

or equivalently,

$$y(k) = \sum_{i=-\infty}^{\infty} x(k-i) h_f(i) \quad (A.4)$$

Substituting equation A.4 into A.2 and squaring we have,

$$\begin{aligned} e^2(k) = & d^2(k) - 2d(k) \sum_{i=-\infty}^{\infty} h_f(i) x(k-i) \\ & + \left[ \sum_{i=-\infty}^{\infty} x(i) h_f(k-i) \right] \left[ \sum_{i=-\infty}^{\infty} x(i) h_f(k-i) \right] \end{aligned} \quad (A.5)$$



Taking the expected value of A.5 we obtain,

$$\begin{aligned}
 E\{e^2(k)\} &= \psi_{dd}(0) - 2 \sum_{i=-\infty}^{\infty} h_f(i) \psi_{dx}(i) \\
 &+ \sum_{i=-\infty}^{\infty} h_f(i) \sum_{k=-\infty}^{\infty} h_f(k) \psi_{xx}(k-i)
 \end{aligned} \tag{A.6}$$

where notation  $\psi_{uv}(m)$  denotes the expected value of the product of  $u(k)$  and  $v(k+m)$ . That is,

$$\psi_{uv}(m) = E\{u(k)v(k+m)\} = E\{u(k)v(k-m)\} \tag{A.7}$$

otherwise known as the autocorrelation of  $u(k)$  and  $v(k)$ .

Using variational techniques and letting,

$$h_f(i) = h_{opt}(i) + \epsilon \Delta h(i) \tag{A.8}$$

substituting equation A.8 into equation A.6, taking the partial derivative with respect to epsilon,  $\epsilon$ , and setting the partial derivative equal to zero we have:

$$\sum_{n=-\infty}^{\infty} \Delta h(n) \left[ \sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) - \psi_{xd}(n) \right] = 0 \tag{A.9}$$

The term within the brackets is the discrete form of the Weiner-Hopf equation and must be equal to zero if equation A.9 is to be valid since  $\Delta h(n) > 0$  by definition. Therefore,

$$\sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \tag{A.10}$$



Equation A.10 is the most often encountered in truncated form in linear prediction theory, Pade approximation, adaptive filtering and lattice filtering. The truncated version of A.10 which is generally used in solving the discrete Weiner problem is,

$$\sum_{i=0}^M h_{\text{opt}}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \quad (\text{A.11})$$

or in matrix form,

$$\begin{bmatrix} \psi_{xx}(0) & \psi_{xx}(-1) & \dots & \psi_{xx}(-n) \\ \psi_{xx}(1) & \psi_{xx}(0) & \dots & \psi_{xx}(1-n) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \cdot & \vdots \\ \vdots & \vdots & & \vdots \\ \psi_{xx}(n) & \psi_{xx}(n-1) & \dots & \psi_{xx}(0) \end{bmatrix} \begin{bmatrix} h_{\text{opt}}(0) \\ h_{\text{opt}}(1) \\ \vdots \\ \vdots \\ \vdots \\ h_{\text{opt}}(n) \end{bmatrix} = \begin{bmatrix} \psi_{xd}(0) \\ \psi_{xd}(1) \\ \vdots \\ \vdots \\ \vdots \\ \psi_{xd}(n) \end{bmatrix} \quad (\text{A.12})$$



# APPENDIX B

```

PROGRAM ADMA: ADJUTRESSIVE-MOVING AVERAGE SIMULATOR
      THIS PROGRAM GENERATES THE VARIOUS TYPES OF LINEAR SYNTHETIC
      DATA WHICH IS PROCESSED BY THE ADAPTIVE KALMAN IDENTIFIER (AKI).
      THE PROGRAM PERFORMS ALL COMPUTATIONS IN DOUBLE PRECISION AND
      SINKS THE OUTPUT DATA ON A DISK FILE FOR ANALYSIS BY THE AKI. THE
      ENDERS OF THE ADJUTRESSIVE (AR) AND MOVING-AVERAGE (MA) PRICE SE-
      LES PRECEDE THE DATA IN UNCOMPRESSED FORM.
      REAL*8 Y(8000),U(8000),Z(8000),V(8000)
      REAL*4 W(8000),W2(8000)
      WRITE (6,100)
100  FORMAT (1F1,1X,4H TIME,10X,5H INPUT,9X,6H MULTIPLY,4X,11H MEASUREMENT,/)
      D=1.0-92545117.00
      SIGV = 1.0E-3
      SIGW = 1.0E-3
      NP= NUMBER OF DATA POINTS GENERATED
      NF= NUMBER OF AN COEFFICIENTS
      ME= NUMBER OF MA COEFFICIENTS
      NI= MODE OF OPERATION FOR THE AKI
      NCI= ICIAL ORDER OF SYSTEM
      NP=400
      ME=2
      NI=0
      NCI=0
      WRITE (6,101) NP,NL
101  FORMAT (1F1,1X,4H TIME,10X,5H INPUT,9X,6H MULTIPLY,4X,11H MEASUREMENT,/)
      CALL GCMPL (DSET,NP,N)
      CALL GCMPL (DSET,NP,W2)
      DO I=1,NP
      V(I) = SIGV*DBLE(W2(I))
      U(I) = DBLE(W(I))
      A1=0.000
      A2=0.000
      A3=0.000
      A4=0.000
      A5=1.000
      A6=5.000
      A7=5.000
      A8=0.000

```





ARM C0520  
ARM C0530  
ARM C0540  
ARM C0550  
ARM C0560  
ARM C0570  
ARM C0580  
ARM C0590  
ARM C0600  
ARM C0610  
ARM C0620  
ARM C0630  
ARM C0640  
ARM C0650  
ARM C0660  
ARM C0670  
ARM C0680  
ARM C0690  
ARM C0700  
ARM C0710  
ARM C0720  
ARM C0730  
ARM C0740  
ARM C0750  
ARM C0760  
ARM C0770  
ARM C0780

```

      AY=0.0000
      ALU=C.0000
      SET THE INITIAL CONDITIONS
      DO 40 I=1,NUT
      Y(I) = 0.0000
      ZC U(I) = 0.0000
      KK=0
      DO 30 K=KUT1,NP
      Y(K)=A1*Y(K-1)+A2*Y(K-2)+A3*Y(K-3)+A4*Y(K-4)+A5*U(K)+
      LAO*U(K-1)+A7*U(K-2)+A8*U(K-3)+A9*U(K-4)+A10*U(K-5)
      Z(K) = Y(K)+V(K)
      KK = KK+1
      DO 20 WKITL (0,200) KK,U(K),Y(K),Z(K)
      ZCC FORMAT (15,4E15.4)
      DO 40 I=1,NP
      X1=SNGL(U(I))
      X2=SNGL(Z(I))
      40 WKITF (2) X1,X2
      STOP
      END

```



## 126

[illegible]



```

10 DEFINE THE # OF PARAMETER STUDIES
11 JKOR=1
12
13 CONVERT THE INPUTS TO RATIANS AND SET THE POWER LEVEL
14 DO 100 J=1,NP
15   W(J) = ((PI*W(J))/180.0)*51CW
16
17 BEGIN MAIN ITERATION LOOP
18 DO 1000 I=1,JKOR
19
20   DEFINE INPUT X, AS A GAUSSIAN INPUT SEQUENCE
21   DO 100 J=1,NP
22     X(J)=DELE(W(J))
23
24   BEGIN THE SIMULATION PHASE LOCKED LOOP
25   CALL NEWGN(PG,C,IG,MCRD,X,Y,NP,10)
26
27   PRINT OUT THE RESULTS OF THE SIMULATION
28   WRITE(CP,ZC)
29   FIRST ATTEMPT, I=1, I=1M0.....INPUT.....OUTPUT.....,
30   I=1, I=2, I=3, I=4, I=5, I=6, I=7, I=8, I=9, I=10, I=11, I=12, I=13, I=14, I=15, I=16, I=17, I=18, I=19, I=20, I=21, I=22, I=23, I=24, I=25, I=26, I=27, I=28, I=29, I=30, I=31, I=32, I=33, I=34, I=35, I=36, I=37, I=38, I=39, I=40, I=41, I=42, I=43, I=44, I=45, I=46, I=47, I=48, I=49, I=50, I=51, I=52, I=53, I=54, I=55, I=56, I=57, I=58, I=59, I=60, I=61, I=62, I=63, I=64, I=65, I=66, I=67, I=68, I=69, I=70, I=71, I=72, I=73, I=74, I=75, I=76, I=77, I=78, I=79, I=80, I=81, I=82, I=83, I=84, I=85, I=86, I=87, I=88, I=89, I=90, I=91, I=92, I=93, I=94, I=95, I=96, I=97, I=98, I=99, I=100, I=101, I=102, I=103, I=104, I=105, I=106, I=107, I=108, I=109, I=110, I=111, I=112, I=113, I=114, I=115, I=116, I=117, I=118, I=119, I=120, I=121, I=122, I=123, I=124, I=125, I=126, I=127, I=128, I=129, I=130, I=131, I=132, I=133, I=134, I=135, I=136, I=137, I=138, I=139, I=140, I=141, I=142, I=143, I=144, I=145, I=146, I=147, I=148, I=149, I=150, I=151, I=152, I=153, I=154, I=155, I=156, I=157, I=158, I=159, I=160, I=161, I=162, I=163, I=164, I=165, I=166, I=167, I=168, I=169, I=170, I=171, I=172, I=173, I=174, I=175, I=176, I=177, I=178, I=179, I=180, I=181, I=182, I=183, I=184, I=185, I=186, I=187, I=188, I=189, I=190, I=191, I=192, I=193, I=194, I=195, I=196, I=197, I=198, I=199, I=200, I=201, I=202, I=203, I=204, I=205, I=206, I=207, I=208, I=209, I=210, I=211, I=212, I=213, I=214, I=215, I=216, I=217, I=218, I=219, I=220, I=221, I=222, I=223, I=224, I=225, I=226, I=227, I=228, I=229, I=230, I=231, I=232, I=233, I=234, I=235, I=236, I=237, I=238, I=239, I=240, I=241, I=242, I=243, I=244, I=245, I=246, I=247, I=248, I=249, I=250, I=251, I=252, I=253, I=254, I=255, I=256, I=257, I=258, I=259, I=260, I=261, I=262, I=263, I=264, I=265, I=266, I=267, I=268, I=269, I=270, I=271, I=272, I=273, I=274, I=275, I=276, I=277, I=278, I=279, I=280, I=281, I=282, I=283, I=284, I=285, I=286, I=287, I=288, I=289, I=290, I=291, I=292, I=293, I=294, I=295, I=296, I=297, I=298, I=299, I=300, 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```









```

5      APPLY SCALE FACTOR TO ARRAY
6      DO 75 I=2,I0
7      DO 75 I=1,MORD
8      APPLY(I,I)=ARRAY(I,I)*PG(I-1,I)
9
10     WHILE(O,I0)
11     FORNAT(7,' THE SCALED COEFFICIENTS ARE ')
12     WRITE(O,I0)
13     WRITE(E,I0) ((ARRAY(I,I),I=1,I0),I=1,MLEP)
14
15     LA=0
16     DO 65 I=1,I0
17     DO 65 I=1,MORD
18     LA=LA+1
19     CSCALE(LA,I)=ARRAY(I,I)
20
21     INITIALIZE FILTER STATES TO ZERO
22     DO 100 I=1,I0
23     DO 100 L=1,LORD
24     F(I,I)=0.000
25
26     CALCULATE NE POINTS OF OUTPUT Y(INS)
27     DO 1000 INS=1,IMP
28
29     DECLARE THE FILTER AT INSTANCE INS
30     F(1,I)=Y(INS)
31     F(2,I)=Y(INS)**2
32     F(3,I)=X(INS)**2
33     F(4,I)=X(INS)*Y(INS)
34     F(5,I)=X(INS)**3
35     F(6,I)=X(INS)*Y(INS)**2
36     F(7,I)=Y(INS)*X(INS)**2
37     F(8,I)=X(INS)**3
38
39     F(1,AC,INS,I)=F(I,I)
40     WHILE(O,I0)
41     FORNAT(7,' THE FILTER STATES ARE ')
42     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
43     FORNAT(7,' THE LOG VECTOR ')
44     WRITE(O,I0)
45
46     NEW VECTOR FILTER STATES INTO DUMMY
47     FORNAT(7,' THE LOG VECTOR ')
48     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
49
50     NEW VECTOR FILTER STATES INTO DUMMY
51     FORNAT(7,' THE LOG VECTOR ')
52     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
53
54     NEW VECTOR FILTER STATES INTO DUMMY
55     FORNAT(7,' THE LOG VECTOR ')
56     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
57
58     NEW VECTOR FILTER STATES INTO DUMMY
59     FORNAT(7,' THE LOG VECTOR ')
60     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
61
62     NEW VECTOR FILTER STATES INTO DUMMY
63     FORNAT(7,' THE LOG VECTOR ')
64     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
65
66     NEW VECTOR FILTER STATES INTO DUMMY
67     FORNAT(7,' THE LOG VECTOR ')
68     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
69
70     NEW VECTOR FILTER STATES INTO DUMMY
71     FORNAT(7,' THE LOG VECTOR ')
72     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
73
74     NEW VECTOR FILTER STATES INTO DUMMY
75     FORNAT(7,' THE LOG VECTOR ')
76     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
77
78     NEW VECTOR FILTER STATES INTO DUMMY
79     FORNAT(7,' THE LOG VECTOR ')
80     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
81
82     NEW VECTOR FILTER STATES INTO DUMMY
83     FORNAT(7,' THE LOG VECTOR ')
84     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
85
86     NEW VECTOR FILTER STATES INTO DUMMY
87     FORNAT(7,' THE LOG VECTOR ')
88     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
89
90     NEW VECTOR FILTER STATES INTO DUMMY
91     FORNAT(7,' THE LOG VECTOR ')
92     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
93
94     NEW VECTOR FILTER STATES INTO DUMMY
95     FORNAT(7,' THE LOG VECTOR ')
96     WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)
97
98     NEW VECTOR FILTER STATES INTO DUMMY
99     FORNAT(7,' THE LOG VECTOR ')
100    WRITE(O,I0) ((F(I,I),I=1,I0),I=1,I0)

```







# APPENDIX D

PROGRAM ALAPISN: ADAPTIVE KALMAN IDENTIFIER (AKI)

THIS PROGRAM REPRESENTS THE MAIN SOFTWARE DEVELOPED WHICH IMPLEMENTS THE ADAPTIVE KALMAN IDENTIFIER (AKI) FORMULATED IN THE BODY OF THE THESIS. THE ADAPTIVE KALMAN IDENTIFIER (AKI) IDENTIFIES THE COEFFICIENTS OF THE GENERAL AUTOREGRESSIVE MOVING-AVERAGE (GAMA) REPRESENTATION OF AN UNKNOWN SYSTEM. HERETOFORE, UNKNOWN SYSTEM PARAMETERS WERE SIMPLY AUGMENTED TO THE STATE VECTOR OF THE KALMAN FILTER AND, HENCE, ESTIMATED. USING THE AKI PROCEDURE, SUCH AUGMENTATION OF EXTENSION IS MADE; BUT, KATFER, THE PARAMETERS OF THE SYSTEM (OR EQUIVALENTLY THE COEFFICIENTS OF THE ARMA EQUATION) ARE IDENTIFIED DIRECTLY.

IDENTIFYING THE ARMA COEFFICIENTS REQUIRES PROCESSING THE INPUT/OUTPUT DATA WHICH IS TEMPORARILY STORED ON A DISK FILE PRIOR TO BEING READ BY THE AKI PROGRAM. THE LEVELS OF THE AUTOREGRESSIVE (AR) AND MOVING-AVERAGE (MA) PROCESS AND THE CHIEF FLAG PRECEDE THE UNFORMATTED DATA ALSO IN UNFORMATTED FORM. THE AKI PROGRAM HAS SEVERAL MULTIPURPOSE FLAGS WHICH ARE DESCRIBED THROUGHOUT THE SOURCE CODE IN THE FORM OF COMMENT STATEMENTS.

```
REAL*8 A(PK,3,H),PKK,PARMI,U,Y,C,FI,Z,R,LU,YY,CY,YU,UG
COMMON A(PK(20,300)),C(20,300),I(20,20),PKK(20,20),PRKAL(20,20)
1,00(800),YY(800),CY(800),YL(800),GG(20,800)
2,J(800),Y(800),C(20,20),EI(20,20),Z,R,N,K,M,NTOT,KK,LDIAG,NI
3,10,11
```

```
REAL*8 SLOW
REAL*4 V1(300),V2(800),A(20),B(20)
COMPLEX*16 L(20),F2(20)
INTEGER N,M,OP,NTOT,LDIAG,IPL,IFOUT,IIR,IPT,NI
```

```
NEWID=0
NEWINC=0
NEWID=0
```

```
IP=0 DATA (INPUT/OUTPUT) PRINTS
IP=300
IPNT=IP-100
```

```
IPL=0 A PLOTTING FLAG; IF IPL=1 SUBROUTINE GRAPH IS CALLED
LDIAG=0 A DIAGNOSTIC FLAG; IF LDIAG=1 MATRIX INTERMEDIATE
VALUES ARE PRINTED OUT; IF IIR=1 A RESULT CALCULATION FLAG,
IF IFOUT=1 INTERMEDIATE VALUES FOR THE RESULTS ARE COMPUTED.
IPT=0 A PRINTING FLAG; PRINTING OUTPUT RESULTS FOR K>IPT.
```

```
IPL=1
LDIAG=0
IFOUT=1
```









```

1 READ IN THE INPUT/OUTPUT DATA OF THE UNKNOWN PLANT TO
2 BE IDENTIFIED
3
4 IF N4 IDENTIFICATION ALA BOX-JENKINS, READ IN Y(K) AND
5 ZERO OUT ALL U(K)
6 I=NP+2*NTOT
7 DO 21 I=1,NP
8   U(I)=C.CDD
9   Y(I)=EDEL(V2(I))
10  GO TO 24
21 CONTINUE
22 READ (8) V2(I)
23
24 CONVERT OUTPUT DATA TO DOUBLE PRECISION
25 DO 23 I=1,NP
26   U(I)=C.CDD
27   Y(I)=EDEL(V2(I))
28 GO TO 24
29 CONTINUE
30
31 READ (8) V1(I),V2(I)
32
33 CONVERT THE INPUT/OUTPUT DATA INTO DOUBLE PRECISION
34 DO 30 I=1,NP
35   U(I)=EDEL(V1(I+NTOT))
36   Y(I)=EDEL(V2(I+NTOT))
37 GO TO 24
38 CONTINUE
39
40 FLAG, THE REQUIRED NON-LINEAR INPUTS FOR THE AL-ANJ
41 IF (NL.EQ.0) GO TO 25
42
43 DO 25 I=1,NP
44   U(I)=C(I)*3
45 IF (NL.EQ.1) GO TO 29
46
47 DO 26 I=1,NP
48   Y(I)=Y(I)*3
49 IF (NL.EQ.2) GO TO 29
50
51 DO 27 I=1,NP
52   U(I)=U(I)**2)*Y(I)
53 IF (NL.EQ.3) GO TO 29
54
55 DO 28 I=1,NP
56   Y(I)=(Y(I)*3)*U(I)
57 IF (NL.EQ.4) GO TO 29
58 CONTINUE
59

```

```

ALAC1000
ALAC1010
ALAC1020
ALAC1030
ALAC1040
ALAC1050
ALAC1060
ALAC1070
ALAC1080
ALAC1090
ALAC1100
ALAC1110
ALAC1120
ALAC1130
ALAC1140
ALAC1150
ALAC1160
ALAC1170
ALAC1180
ALAC1190
ALAC1200
ALAC1210
ALAC1220
ALAC1230
ALAC1240
ALAC1250
ALAC1260
ALAC1270
ALAC1280
ALAC1290
ALAC1300
ALAC1310
ALAC1320
ALAC1330
ALAC1340
ALAC1350
ALAC1360
ALAC1370
ALAC1380
ALAC1390
ALAC1400
ALAC1410
ALAC1420
ALAC1430
ALAC1440
ALAC1450
ALAC1460
ALAC1470

```



```

10      LOAD THE INITIAL VALUES OF THE MEASUREMENT VECTOR
11      IF (N.EQ.0) GO TO 40
12      LN = LN+1
13
14      LOAD THE MEASUREMENT VECTOR WITH Y(K-1) ELEMENTS
15      DO 30 I=1,N
16      L = LN-I
17      H(L,L) = Y(1+N(L)-N-1)
18
19
20
21      30 CONTINUE
22      IF (N.EQ.0) GO TO 60
23      MN = MN+1
24
25      LOAD THE MEASUREMENT VECTOR WITH U(K-1) ELEMENTS
26      DO 50 I=1,M
27      LL = MN-I
28      H(LL,LL+N) = U(1+N(LL)-M)
29
30
31      IF (N.EQ.0).OR.(N.EQ.5) GO TO 60
32
33      LOAD THE MEASUREMENT VECTOR WITH NON-LINEAR ELEMENTS U(K-1)*3
34      IF (N.EQ.0) GO TO 53
35      MA = MN+1
36      DO 50 I=1,M
37      LI = MN-I
38      H(LI,LI+NM) = UU(1+N(LLI)-M)
39
40
41      50 IF (N.EQ.1) GO TO 60
42
43      LOAD THE MEASUREMENT VECTOR WITH NON-LINEAR ELEMENTS Y(K-1)*3
44      IF (N.EQ.3) GO TO 5,
45      LN = LN+1
46      L2 = L+1
47      L2 = L2-1
48      H(L1,L2+M+2*3) = YY(1+N(LLI)-N-1)
49
50
51      55 IF (N.EQ.2) GO TO 6,
52
53      LOAD THE MEASUREMENT VECTOR WITH NON-LINEAR ELEMENTS Y(K)*U(K)*2
54      IF (N.EQ.0).OR.(N.EQ.3) GO TO 60
55      LN = LN+1
56      L3 = LN
57      L3 = LN-1
58      H(L1,L3+2*3+2*3M) = UY(1+N(LLI)-N-1)
59
60
61      IF (N.EQ.3) GO TO 62
62
63      LOAD THE MEASUREMENT VECTOR WITH NON-LINEAR ELEMENTS U(K)*Y(K)*2
64      LN = LN+1
65      DO 67 I=1,N

```







```

DO 21 J=1,NLCL
  21 WRITE (6,22) (Q(I,J),J=1,NLCL)
  22 FORMAT (10E12.4)
C
C MAIN MAIN ITERATION LOOP
C
NN=0
NOLNKF = N+1
DO 30 K=RS1701,IP
  KK = NN+1
  C
  C COMPUTE THE GAINS, C(K), AND FREQU COVARIANCES, PRR(K) & PRNMI(F)
  C CALL CALN
  C
  C COMPUTE ESTIMATE OF THE ARMA COEFFICIENTS
  C CALL REGRN
  C
  C ADAPTIVE KALMAN ARMA COEFFICIENTS ARE NOW COMPUTE FOR
  C THE STEP, N, AND ARE READY FOR PRINTING
  C
  IF (K.EQ.IPST) GO TO 100
  CALL PRNT
  C
  IF (KROU.LS.O) GO TO 100
  C
  C COMPUTE THE POLES AND ZEROS
  M1=M-1
  IF (M1.LS.O) GO TO 170
  M1=M+1
  DO 120 L=1,M,NLCL
    120 A(1-M1+L) = SGL(AHKK(1,K+1))
  C
  CALL ZERFLY (A,1L,FL,IER)
  C
  WRITE (6,130)
  130 FORMAT (/5X,10HDIFF ZEROS: /,5X,ZHRT,9X,ZH1P,/)
  140 A(1L,C,140) = 1(1),1=1,M1)
  140 WRITE (6,140)
  150 WRITE (6,140) IER
  170 CONTINUE
  C
  IF (N.EQ.O) GO TO 100
  L(1) = 1.0
  DO 140 L=1, .
  140 P(141) = -SGL(CAPK(1,K+1))
  C
  CALL ZERFLY (O,4,K2,IER)

```

```

A1A(2440
A1A(2450
A1A(2460
A1A(2470
A1A(2480
A1A(2490
A1A(2500
A1A(2510
A1A(2520
A1A(2530
A1A(2540
A1A(2550
A1A(2560
A1A(2570
A1A(2580
A1A(2590
A1A(2600
A1A(2610
A1A(2620
A1A(2630
A1A(2640
A1A(2650
A1A(2660
A1A(2670
A1A(2680
A1A(2690
A1A(2700
A1A(2710
A1A(2720
A1A(2730
A1A(2740
A1A(2750
A1A(2760
A1A(2770
A1A(2780
A1A(2790
A1A(2800
A1A(2810
A1A(2820
A1A(2830
A1A(2840
A1A(2850
A1A(2860
A1A(2870
A1A(2880
A1A(2890
A1A(2900
A1A(2910

```

















```

DOUBROVINE BLOCKS,
THIS SUBROUTINE COMPUTES AN ESTIMATE FOR THE ADAPTIVE
AREA ELEMENTS
C
C
C
      REAL*8 ATRK(5,5),PRK,PRKM1,U,Y,C,F,I,Z,N,UU,YY,UY,YU,GG
      COMMON ATRK(20,800),G(20,800),F(20,20),PRK(20,20),PRKM1(20,20)
      UO(800),YI(800),UY(800),YU(800),GG(20,800)
      Z,U(800),Y(800),G(20,20),F(20,20),Z,R,N,K,M,N,NFOL,KK,IOLAG,NL
      3,NOL,I
      REAL*8 EL,TEMPA(20,20),TEMPB(20,20)
      COMMON ATRK(I,I+1) = ATRK(K(K)+G(N+1))(Z-H(K))ATRK(K(K))
C
C
      DO 10 I=1,NFOL
      DO 10 J=1,NFOL
      TEMPA(I,J) = 0.0000
      TEMPB(I,J) = 0.0000
      10 IF (J.GE.1) TEMPA(I,J) = ATRK(I,K)
C
      IF (TEMPA(20,0)0) GO TO 40
      WRITE (6,11)
      11 FORNAT (7,5X,'I4 RECKEN, TEMPA:',/)
      DO 112 I=1,NFOL
      112 WRITE (6,113) (TEMPA(I,J),J=1,NFOL)
      113 FORNAT (10,12.4)
      WRITE (6,114)
      115 FORNAT (7,5X,'I4 RECKEN, H(K):',/)
      DO 117 I=1,NFOL
      117 WRITE (6,117) (H(I,J),J=1,NFOL)
      40 CONTINUE
C
      CALL PRCL (H,TEMPA,NFOL,NFOL,TEMPB)
      Z = Y(K)
C
      IF (TEMPB(20,0)0) GO TO 30
      WRITE (6,11)
      114 FORNAT (7,5X,'I4 RECKEN, H(K)*ATRK(K):',/)
      DO 115 I=1,NFOL
      115 WRITE (6,116) (TEMPB(I,J),J=1,NFOL)
      116 FORNAT (10,12.4)
      WRITE (6,117)
C
      118 Z = TEMPB(1,1)
C
      IF (NOL.NE.0) GO TO 32
      UO(1) = 30
      WRITE (6,31) ID,TEMPB(1,1)
      31 FORNAT (7,5X,'TEMPER(NOL):',5X,1F.5,5X,'PRFTEMPB:',5X,1F.5,/)

```

```

AFAC4030
AFAC4100
AFAC4110
AFAC4120
AFAC4130
AFAC4140
AFAC4150
AFAC4160
AFAC4170
AFAC4180
AFAC4190
AFAC4200
AFAC4210
AFAC4220
AFAC4230
AFAC4240
AFAC4250
AFAC4260
AFAC4270
AFAC4280
AFAC4290
AFAC4300
AFAC4310
AFAC4320
AFAC4330
AFAC4340
AFAC4350
AFAC4360
AFAC4370
AFAC4380
AFAC4390
AFAC4400
AFAC4410
AFAC4420
AFAC4430
AFAC4440
AFAC4450
AFAC4460
AFAC4470
AFAC4480
AFAC4490
AFAC4500
AFAC4510
AFAC4520
AFAC4530
AFAC4540
AFAC4550

```









```

SUBROUTINE SUBR (C,P,R,N,M,C)
  THIS SUBROUTINE SUBTRACTS TWO KXM MATRICES AND RETURNS THE
  RESULT IN MATRIX C
  INTEGER N,M
  DO 10 I=1,N
    DO 10 J=1,M
      C(I,J) = A(I,J)-B(I,J)
    RETURN
  END
  SUBROUTINE ADD (A,P,R,N,M,C)
  THIS SUBROUTINE ADDS THE KXM MATRICES A AND B AND RETURNS THE
  RESULT IN MATRIX C
  INTEGER N,M
  DO 10 I=1,N
    DO 10 J=1,M
      C(I,J) = A(I,J) + B(I,J)
    RETURN
  END

```

```

A(5,5)=0
A(5,6)=0
A(6,5)=0
A(6,6)=0
A(7,5)=0
A(7,6)=0
A(8,5)=0
A(8,6)=0
A(9,5)=0
A(9,6)=0
A(10,5)=0
A(10,6)=0
A(11,5)=0
A(11,6)=0
A(12,5)=0
A(12,6)=0
A(13,5)=0
A(13,6)=0
A(14,5)=0
A(14,6)=0
A(15,5)=0
A(15,6)=0
A(16,5)=0
A(16,6)=0
A(17,5)=0
A(17,6)=0
A(18,5)=0
A(18,6)=0
A(19,5)=0
A(19,6)=0
A(20,5)=0
A(20,6)=0
A(21,5)=0
A(21,6)=0
A(22,5)=0
A(22,6)=0
A(23,5)=0
A(23,6)=0
A(24,5)=0
A(24,6)=0
A(25,5)=0
A(25,6)=0
A(26,5)=0
A(26,6)=0
A(27,5)=0
A(27,6)=0
A(28,5)=0
A(28,6)=0
A(29,5)=0
A(29,6)=0
A(30,5)=0
A(30,6)=0
A(31,5)=0
A(31,6)=0
A(32,5)=0
A(32,6)=0
A(33,5)=0
A(33,6)=0
A(34,5)=0
A(34,6)=0
A(35,5)=0
A(35,6)=0
A(36,5)=0
A(36,6)=0
A(37,5)=0
A(37,6)=0
A(38,5)=0
A(38,6)=0
A(39,5)=0
A(39,6)=0
A(40,5)=0
A(40,6)=0
A(41,5)=0
A(41,6)=0
A(42,5)=0
A(42,6)=0
A(43,5)=0
A(43,6)=0
A(44,5)=0
A(44,6)=0
A(45,5)=0
A(45,6)=0
A(46,5)=0
A(46,6)=0
A(47,5)=0
A(47,6)=0
A(48,5)=0
A(48,6)=0
A(49,5)=0
A(49,6)=0
A(50,5)=0
A(50,6)=0
A(51,5)=0
A(51,6)=0
A(52,5)=0
A(52,6)=0
A(53,5)=0
A(53,6)=0
A(54,5)=0
A(54,6)=0
A(55,5)=0
A(55,6)=0
A(56,5)=0
A(56,6)=0
A(57,5)=0
A(57,6)=0
A(58,5)=0
A(58,6)=0
A(59,5)=0
A(59,6)=0
A(60,5)=0
A(60,6)=0
A(61,5)=0
A(61,6)=0
A(62,5)=0
A(62,6)=0
A(63,5)=0
A(63,6)=0
A(64,5)=0
A(64,6)=0
A(65,5)=0
A(65,6)=0
A(66,5)=0
A(66,6)=0
A(67,5)=0
A(67,6)=0
A(68,5)=0
A(68,6)=0
A(69,5)=0
A(69,6)=0
A(70,5)=0
A(70,6)=0
A(71,5)=0
A(71,6)=0
A(72,5)=0
A(72,6)=0
A(73,5)=0
A(73,6)=0
A(74,5)=0
A(74,6)=0
A(75,5)=0
A(75,6)=0
A(76,5)=0
A(76,6)=0
A(77,5)=0
A(77,6)=0
A(78,5)=0
A(78,6)=0
A(79,5)=0
A(79,6)=0
A(80,5)=0
A(80,6)=0
A(81,5)=0
A(81,6)=0
A(82,5)=0
A(82,6)=0
A(83,5)=0
A(83,6)=0
A(84,5)=0
A(84,6)=0
A(85,5)=0
A(85,6)=0
A(86,5)=0
A(86,6)=0
A(87,5)=0
A(87,6)=0
A(88,5)=0
A(88,6)=0
A(89,5)=0
A(89,6)=0
A(90,5)=0
A(90,6)=0
A(91,5)=0
A(91,6)=0
A(92,5)=0
A(92,6)=0
A(93,5)=0
A(93,6)=0
A(94,5)=0
A(94,6)=0
A(95,5)=0
A(95,6)=0
A(96,5)=0
A(96,6)=0
A(97,5)=0
A(97,6)=0
A(98,5)=0
A(98,6)=0
A(99,5)=0
A(99,6)=0
A(100,5)=0
A(100,6)=0

```







[illegible]





```

100 IF (N.E.C.1) GO TO 40
      SUBROUTINE OUTPUT IN DATA
10  IF (N.E.C.) GO TO 100
11  IF (N.E.C.1) GO TO 110
      L1=N+1
      L2=N+1
      L3=N+2*W
      DO 120 I=1,L1
      L4=I-1
120 IF (1,L1+L3) = 0(1,L1+L3-1)
130 IF (1,L3+1) = Y(Y)
100 IF (N.E.C.2) GO TO 40
      SUBROUTINE CROSS INPUT/OUTPUT IN DATA
10  IF (N.E.C.) GO TO 100
11  IF (N.E.C.1) GO TO 110
      L1=N+1
      L2=N+1
      L3=I*N+2*W
      DO 130 I=1,L2
      L4=I-1
130 IF (1,L1+L3) = 0(1,L1+L3-1)
140 IF (1,L3+1) = 0(1,N)
100 IF (N.E.C.3) GO TO 40
      SUBROUTINE CROSS OUTPUT/INPUT IN DATA
10  IF (N.E.C.1) GO TO 100
      L1=N+1
      L2=N+1
      L3=I*N+2*W
      DO 150 I=1,L2
      L4=I-1
150 IF (1,L1+L3) = 0(1,L1+L3-1)
100 IF (1,L3+1) = Y(Y)
      GO TO 100
      IF (N.E.C.4)
      IF (N.E.C.5)
      IF (N.E.C.6)
      IF (N.E.C.7)
      IF (N.E.C.8)
      IF (N.E.C.9)
      IF (N.E.C.10)
      IF (N.E.C.11)
      IF (N.E.C.12)
      IF (N.E.C.13)
      IF (N.E.C.14)
      IF (N.E.C.15)
      IF (N.E.C.16)
      IF (N.E.C.17)
      IF (N.E.C.18)
      IF (N.E.C.19)
      IF (N.E.C.20)
      IF (N.E.C.21)
      IF (N.E.C.22)
      IF (N.E.C.23)
      IF (N.E.C.24)
      IF (N.E.C.25)
      IF (N.E.C.26)
      IF (N.E.C.27)
      IF (N.E.C.28)
      IF (N.E.C.29)
      IF (N.E.C.30)
      IF (N.E.C.31)
      IF (N.E.C.32)
      IF (N.E.C.33)
      IF (N.E.C.34)
      IF (N.E.C.35)
      IF (N.E.C.36)
      IF (N.E.C.37)
      IF (N.E.C.38)
      IF (N.E.C.39)
      IF (N.E.C.40)
      IF (N.E.C.41)
      IF (N.E.C.42)
      IF (N.E.C.43)
      IF (N.E.C.44)
      IF (N.E.C.45)
      IF (N.E.C.46)
      IF (N.E.C.47)
      IF (N.E.C.48)
      IF (N.E.C.49)
      IF (N.E.C.50)
      IF (N.E.C.51)
      IF (N.E.C.52)
      IF (N.E.C.53)
      IF (N.E.C.54)
      IF (N.E.C.55)
      IF (N.E.C.56)
      IF (N.E.C.57)
      IF (N.E.C.58)
      IF (N.E.C.59)
      IF (N.E.C.60)
      IF (N.E.C.61)
      IF (N.E.C.62)
      IF (N.E.C.63)
      IF (N.E.C.64)
      IF (N.E.C.65)
      IF (N.E.C.66)
      IF (N.E.C.67)
      IF (N.E.C.68)
      IF (N.E.C.69)
      IF (N.E.C.70)
      IF (N.E.C.71)
      IF (N.E.C.72)
      IF (N.E.C.73)
      IF (N.E.C.74)
      IF (N.E.C.75)
      IF (N.E.C.76)
      IF (N.E.C.77)
      IF (N.E.C.78)
      IF (N.E.C.79)
      IF (N.E.C.80)
      IF (N.E.C.81)
      IF (N.E.C.82)
      IF (N.E.C.83)
      IF (N.E.C.84)
      IF (N.E.C.85)
      IF (N.E.C.86)
      IF (N.E.C.87)
      IF (N.E.C.88)
      IF (N.E.C.89)
      IF (N.E.C.90)
      IF (N.E.C.91)
      IF (N.E.C.92)
      IF (N.E.C.93)
      IF (N.E.C.94)
      IF (N.E.C.95)
      IF (N.E.C.96)
      IF (N.E.C.97)
      IF (N.E.C.98)
      IF (N.E.C.99)
      IF (N.E.C.100)

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COPUTAL GAMP, (KP)
INFO OUTC(OUTC) SLEPLY PRINTS OUT THE VARIABLES ADRK AND
O(R) IN ORDERLY FLEEM FOR PLOTTING PURPOSES

      K=180  Z=FK, G=H, PKK, PKML, D, Y, G, U, Z, R, LU, Y, LY, ZU, CG
COPUTC, ADRK(20,800), G(20,800), F(20,20), FKK(20,20), PRN 41(20,20)
1, DU(800), Y, Y(800), LY(800), YL(800), G(20, ECC)
2, U(800), Y(800), Z(20,20), F(20,20), Z, R, N, K, Y, NLU, KK, LDIAG, KL
3, LU, LI
      N=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 79
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# APPENDIX E

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PROGRAM LAS: WIDROW LEAST MEAN SQUARES FILTER
      THIS PROGRAM COMPUTES THE MOVING-AVERAGE, MA, (OR EQUIVALENTLY
      THE TRANSVERSAL FILTER WEIGHTS) COEFFICIENTS OF A LEAST MEANS
      SQUARES FILTER USING AN ADAPTIVE ALGORITHM WHICH IMPLEMENTS THE
      METHOD OF STEEPEST DESCENT. THIS ALGORITHM WAS FIRST DEvised
      BY WIDROW.
      REAL*8 Y(2000),W(12),X(12),XX(12),U(2000),F,KS,D,C
      REAL*4 V1(2000),V2(2000),WW(12,2000)
      INTEGER M,NP,KK
      THE CONVERGENCE FACTOR IS KS
      KS = -2.0D-1
      WRITE (6,100) KS
      100 FORMAT (1H1,10X,28HTHE CONVERGENCE FACTOR: KS =,F10.4,/)
      THE ORDER OF THE FILTER IS M
      READ (6) K1,M,K2
      200 WRITE (6,200) M
      200 FORMAT (10X,15,2X,21HWEIGHTS ARE ESTIMATED,/)
      THE DIAGNOSTIC FLAG IS IDIAG
      IDIAG = 0
      WRITE (6,300)
      300 FORMAT (1X,11HTIME:,3X,5HERROR,4X,6HOUTPUT,6X,4HW(1),6X,4HW(2),
      1,6X,4HW(3),6X,4HW(4),6X,4HW(5),6X,4HW(6),6X,4HW(7),6X,4HW(8),
      2,6X,4HW(9),5X,5HW(10),/)
      THE NUMBER OF ITERATIONS IS NP
      NP = 100
      DO 10 I=1,M
      X(I) = 0.0D0
      X(I) = 0.0D0
      Y(I) = 0.0D0
      10 XX(I) = 0.0D0
      DO 20 I=1,300
      20 READ (8) V1(I),V2(I)
      DO 30 I=1,NP
      U(I) = DBLE(V1(I+10))
      30 Y(I) = DBLE(V2(I+10))

```



```

C      BEGIN RECURSIVE ITERATION LOOP
C      DO 40 K=2,NP
C      U = Y(N-1)
C
C      IF (L1LAG.EQ.0) GO TO 80
C      WRITE (6,500) D
C      100 FORMAT (7,5X,'DESIRED SIGNAL D(K):',5X,E15.5,/)
C      80 CONTINUE
C
C      COMPUTE ERROR
C      IF (L1LAG.EQ.0) GO TO 110
C      WRITE (6,120)
C      120 FORMAT (7,5X,'W(I):',10X,'X(I):',/)
C      DO 130 I=1,M
C      130 WRITE (6,140) W(I),X(I)
C      140 FORMAT (5X,2F15.4)
C      110 CONTINUE
C
C      CALL RMULT1(W,X,1,C)
C      E=U-C
C
C      1400 WRITE (6,400) K,E,C,(W(I),I=1,M)
C      1400 FORMAT (14,12F15.3)
C
C      KL=K-1
C      DO 150 L=1,M
C      150 W(L,KL) = SNGL(W(L))
C
C      IF (L1LAG.EQ.0) GO TO 50
C      WRITE (6,500)
C      1500 FORMAT (7,5X,'X(I):',/)
C      1600 WRITE (6,600) (X(I),I=1,M)
C      1600 FORMAT (5X,115.4)
C      50 CONTINUE
C
C      COMPUTE W(K+1): THE NEXT STEP PREDICTION
C      DO 60 I=1,M
C      60 W(I) = KS*(X(I)
C
C      1700 WRITE (6,700) (W(I),I=1,M)
C      1700 FORMAT (5X,115.4)
C      70 CONTINUE
C
C      SUBROUTINE INPUT DATA VECTOR THRU THE DELAY LINE BY ONE
C      L = M+1
C      N = M-1
C      DO 70 I=1,N
C      70 X(LI) = X(LI-1)
C      X(1) = U(K)

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```

C      40 CONTINUE
      NPP=NP-1
      WRITE (4,160) N,NPP
      160 FORMAT(2I5)
C
      DO 170 I=1,M
      DO 170 J=1,NPP
      170 WRITE (4,180) WW(I,J)
      180 FORMAT(4E20.5)
C
      STOP
      END
C

```

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```

SUBROUTINE MULI(B,C,M,D)
REAL*8 B,C,D
DIMENSION B(12),C(12)
INTEGER M
      D=0.000
      DO 10 I=1,M
        D=D+B(I)*C(I)
      10 RETURN
      END

```

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208 FORMAT (/ ,10X,'ONLY THE LAST',I3,' ITERATIONS ARE PRINTED',/)
C
300 WRITE (6,300)
300 FORMAT (9X,'TIME ',10X,5HERROR,5X,6HOUTPUT,/)
C
C
C   THE NUMBER OF ITERATIONS IS NP
NP = 4000
N=N+1
NPRT = NP-1PRT
IF ((M.LT.2).OR.(N.LT.1)) GO TO 205
C
C
DO 10 I=1,M
  A(I) = 0.000
  X(I) = 0.000
  10 XA(I) = 0.000
C
DO 11 I=1,N
  B(I) = 0.000
  11 Y(I) = 0.000
C
DO 20 I=1,NP
  20 READ (8) V1(I),V2(I)
C
NP = NP-10
DO 30 I=1,NP
  U(I) = DBLE(V1(I+10))
  30 Y(I) = DBLE(V2(I+10))
C
BEGIN ADAPTIVE ITERATION LOOP
DO 40 K=2,NP
  U = Y(K-1)
  K3 = K-1
C
  IF (LEIAG.EQ.0) GO TO 80
  WRITE (6,500) K3,D
  500 FORMAT (/ ,5X,'DESIRED SIGNAL D(',I3,'):',5X,F15.5,/)
  80 CONTINUE
C
  COMPUTE ERROR
  IF (LEIAG.EQ.0) GO TO 110
  WRITE (6,122)
  122 FORMAT (/ ,5X,'COMPUTE THE ERROR WITH:',/)
  WRITE (6,120)
  120 FORMAT (/ ,10X,'A(I):',10X,'Y(I):',/)
  DO 130 I=1,N
  130 WRITE (6,140) A(I),X(I)
  140 FORMAT (5X,2F15.4)

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      WRITE (6,121)
121  FORMAT (/,'10X','B(1):',10X,'Y(1):',/,/)
      DO 131 I=1,N
131  WRITE (6,140) B(1),Y1(I)
11) CONTINUE
C
      CALL WMULT (A,X,M,C1)
      CALL MULTS (B,Y1,N,C2)
C
      IF (LDIAG.EQ.0) GO TO 111
      WRITE (6,112) C1,C2
112  FORMAT (/,'5X','A*X=C1=',E15.4,/,5X,'B*Y=C2=',E15.4,/)
111  CONTINUE
C
      Y1(1)=C1+C2
      F=B-Y1(1)
C
      IF (N.GE.NPFI) WRITE (6,403) K8,E,Y1(1)
403  FORMAT (110,2E15.4)
C
      DO 153 L=1,M
150  AA(L,K8) = SNGI(A(L))
      DO 151 L=2,N
151  BB(L-1,K8) = SNGI(B(L))
C
      IF (LDIAG.EQ.0) GO TO 50
      WRITE (6,500)
500  FORMAT (/,'5X','X(1):',/,)
      WRITE (6,600) (X(I),I=1,M)
      WRITE (6,501)
501  FORMAT (/,'5X','Y(1):',/,)
      WRITE (6,600) (Y1(I),I=2,N)
500  FORMAT (5X,E15.5)
50  CONTINUE
C
      COMPUTE A(K+1) AND B(K+1): THE NEXT STEP PREDICTION
      DO 60 I=1,M
      XX(I) = KA*F*X(I)
      A(I) = A(I)-XX(I)
C
      DO 61 I=2,N
      YY(I) = KB*F*Y1(I)
      B(I) = B(I)-YY(I)
C
      IF (LDIAG.EQ.0) GO TO 62
      WRITE (6,63)
63  FORMAT (/,'10X','AFTER UPDATE:',/,10X,'A(1):',/,)

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        WRITE (6,64) (A(I),I=1,M)
64   FORMAT (1X,F14.4)
        WRITE (6,65)
65   FORMAT (/,10X,'AFTER UPDATE:',/,10X,'B(I):',/,)
        WRITE (6,64) (B(I),I=1,N)
62   CONTINUE

C      SHIFT THE INPUT DATA VECTOR THRU THE DELAY LINE BY ONE
C      IF (M.EQ.1) GO TO 72
        L1 = M+1
        N1 = M-1
        DO 70 I=1,N1
        L1 = L1-1
70   X(L1) = X(L1-1)
72   X(L1) = U(K)

        L2 = N+1
        L3 = N-1
        DO 71 I=1,L3
        L4 = L2-I
71   Y1(L4) = Y1(L4-1)

        40   CONTINUE

C      WRITE THE ORDER AND 4 PTS TO OUTPUT FILE FOR GRAPHING
C      NPP=NP-1
        NPP1 = 400
        N5=N-1
        WRITE (4,160) M,NPP1
        WRITE (5,160) N5,NPP1
160   FORMAT(2I5)

        DO 170 I=1,M
        DO 170 J=1,NPP,1)
170   WRITE (4,180) AA(I,J)

        DO 171 I=1,N5
        DO 171 J=1,NPP,1)
171   WRITE (3,180) BB(I,J)
180   FORMAT (4L20.5)

C      WRITE (6,201) (13LA(I),I=1,M)
201   FORMAT (1H1,12(0X,A4),/)

        DO 202 L=NPNT,NPP
202   WRITE (6,203) (AA(I,L),I=1,M)

        203   FORMAT (2A,10E11.3)

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C      WRITE (6,201) (I,BLH(I),I=1,N5)
C      IF 204 L=NPRI,NPP
C      204 WRITE (6,203) (B3(I,L),I=1,N5)
C      GO TO 206
C      205 WRITE (6,207)
C      207 FORMAT (1H1,/,5X,'EITHER THE NUMERATOR OR THE DENOMINATOR ORDER'
C      1, ' IS INCORRECT: THAT IS,/,5X,'EITHER M OR N IS INCORRECTLY'
C      2, ' SPECIFIED. READ IN THE CORRECT',/,5X,'VALUES FOR M AND N AND'
C      3, ' TRY AGAIN.',/)
C      206 CONTINUE
C      STOP
C      END
C

```



```

C
C
C
C
SUBROUTINE KMUL (A,B,N,D)
      REAL*8 A(12),B(12),D
      INTEGER N
      D=0.000
      DO 10 I=1,N
        10 D=L+A(I)*B(I)
      RETURN
      END
C
C

```

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LMS02160
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LMS02270

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```

SUBROUTINE MULT3 (A,B,N,D)
  REAL *8 A(12),B(12),D
  INTEGER N
  D=0.000
  DO 10 I=2,N
    D=D+A(1)*B(1)
  10
  RETURN
END

```

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LMS02300
LMS02310
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LMS02330
LMS02340
LMS02350
LMS02360
LMS02370
LMS02380
LMS02390
LMS02400

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Thesis

M3979 Mayoral

c.1

An Adaptive Kalman  
Identifier and its  
application to linear  
and non-linear ARMA  
modeling.

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